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T3 ________________	Problem Chosen	F3 ________________
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2017
MCM/ICM
Summary Sheet

The contribution made by self-driving cars

Summary

Self-driving cars is a kind of vehicle that can work as a whole, and connect with each other through the cooperating system. In this paper, we investigate the traffic flow control problem from two angles including discrete angle and general angle.

Firstly,we establish a model focusing on specific individuals to explore the question. Precisely, we figure out the maximum traffic density when the speed of the vehicles reaches to the maximum value, as well as the separate traffic density under different percentage of self-driving cars. Regardless of vehicle types, we figure out that when all the vehicles are self-driving, the maximum traffic density is 72 vehicles per mile; when all the traffic vehicles are non-self-driving,the maximum traffic density is 36 vehicles per mile.

Secondly, we refer to the LWR model, $\frac{\partial}{\partial t}\rho(x,t)+\frac{\partial}{\partial x}q(x,t)=0$, to discuss the traffic problem from the angle of traffic flow. Applying Mathematica, we figure out the numerical solution of this equation, and the value of traffic flow by traffic density and speed. Using the relation between speed and traffic density, we get the corresponding maximum number of vehicle, with the percentage of self-driving cars changing. Next, we compared Road carrying capacity with the average daily traffic counts on one lane in one peak hour, which is calculated with the given data in the Excel. After that, we find out that, 70% is the equilibria, that is to say, when the proportion of self-driving car reaches to 70%, each piece of lane would be unblocked; 30% is the tipping point, and when the percentage gets to 30%, the traffic condition would be improved dramatically.

Thirdly, we extended our model, taking the dedicated lane, the curve lane and the gesture of changing lanes into consideration. After calculating,we get the corresponding critical proportion of each road. Taking a piece of road on the 90th route for instance,the start Milepost is 3.94 and endMilepost is 25.37, the critical proportion is 62%, that is to say, when the percentage of self-driving car is less than 62%, there is no need to have a dedicated lane for self-driving cars. Furthermore,under the condition of curve lanes and the gesture of changing lanes,the initial PDE will be explored into $\frac{\partial}{\partial t}\rho(x,t)+\frac{\partial}{\partial x}q(x,t)=f(x,t)$, with f related to specific traffic circumstance.

Finally, we make the sensitivity analysis, and find out that the relationship between the number of lanes,the percentage of self-driving cars and the road carrying capacity is stable.

Contents

1 Introduction

1.1 Background

Looking for ways to optimize the work efficiency has always been attached to significant value, since the very beginning of the human civilization. The introducing of the self-driving cars might bring both positive and negative influence on the conventional traffic model, in which the non-self-driving cars take the dominant place. The typical edges that the self-driving cars have over the non-self-driving cars are the reducing of traffic congestion and traffic accidents, the declining of death and injury rates, the decreasing of related expenses, e.g. the insurance costs, and the increasing of the roadway capacity which might be four times as large as the original one. From the perspective of facilitating people' s daily life, the introducing of self-driving cars might also increase the drivers' extra free time for both leisure and work, and enhance the mobility of the disabled and the senior as well as the low-income citizens.

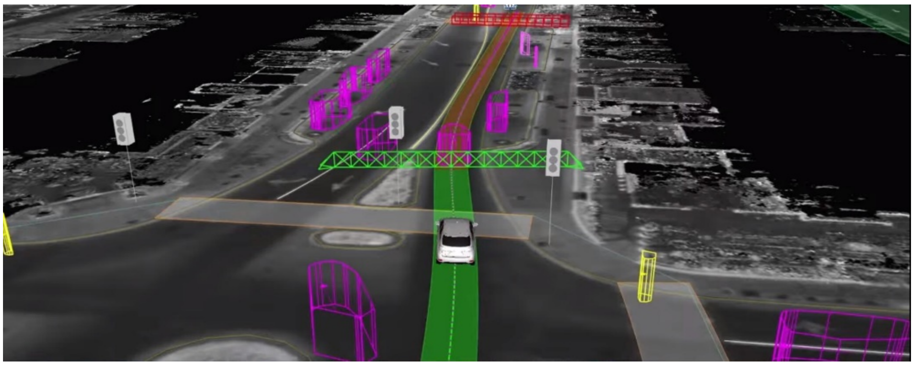

Figure 1: The working mechanism of self-driving cars

1.2 Problem Restatement

The problem that we need to solve in this paper is that:

- Figure out the difference between self-driving cars and non-self-driving car.
- To what extend does the self-driving cars affect the traffic flow and interact with the existing traffic flow?
- How do the effects change as the percentage of self-driving cars increases from 10% to 50% to 90%? Do equilibria and tipping point exist?
- Under what conditions should lanes be spared for self-driving cars in particular?

1.3 Literature Review

Carpool lane is also a kind of lane which is designed to help encourage commuting or sharing cars.The first carpool lanes in Europe were built in the 1990s, but in the US and Canada, they were installed much sooner. California had some lanes dedicated to this purpose by the 1970s, although they were often only "active" during rush hour or peak traffic times. This meant during other times, the lanes were open to anyone who wished to drive in them, no matter whether they had passengers or did not. In addition to allowing carpoolers to use the lanes, they were a frequent choice of buses too, which could quickly speed up trips via bus to various locations.

While it's often most likely people will find these special lanes on an actual freeway or highway, they can be separated from the highway. Some are freestanding. A few are built only for people with multiple passengers.

1.4 Result

In section 3, we made a Discrete Traffic Flow Model and get the functions between speed and traffic density, and figure out the corresponding maximum traffic density when the speed of the vehicles reaches to the maximum value. In section 4, we made a Continuous PDE Model, and refer to the LWR model to discuss the traffic problem from the angle of traffic flow, then we investigate the relationship between the corresponding maximum number of vehicle (Road carrying capacity) that a single period of lane could afford in one hour and the percentage of self-driving cars.

2 Preliminary

2.1 Terms

- Greenshields Model:In 1935, Greenshields put forward a linear model between the speed and traffic density through the statistical analysis of observational data.
- LWR Model: The first major step in macroscopic modeling of traffic was taken by Lighthill and Whitham in 1955, when they indexed the comparability of 'traffic flow on long crowded roads' with 'flood movements in long rivers'. A year later, Richards (1956) complemented the idea with the introduction of ' shock-waves on the highway', completing the so-called LWR model.

2.2 Symbols

Symbols	Descriptions	Unit
q	Traffic flow, the number of vehicles that get through a point in unit time	vehicle per hour
ρ	Traffic density, the number of vehicles that occupy one unit distance.	vehicle per mile
t	Time	h
N	The number of the vehicles occupy one period of distance and a period of time.	
δt	Unit time	h
δx	Unit distance	mile
u	The speed of the vehicles	mile$\cdot h^{-1}$
a	Acceleration, the rate of change of speed of an object with respect to time.	mile$\cdot h^{-2}$
c	braking coefficient	mh^2/km^2
d	the distance from the head of the front car to the head of the selected car.	M

2.3 General assumptions and justifications

- The speed of the vehicles is only related to the traffic density, and according to the Greenshields model, and there was a linear correlation between speed and traffic density.
- The traffic density have similar effects on self-driving, cooperating cars as well as non-self-driving cars.
- It takes 1s for a driver to make a reaction towards the braking made by the front car. There were researches saying that, the time for a human to make reaction after an accident taking place is about 1 second.
- It takes 0.2s for a self-driving car to make response to the braking made by the front car as the response time of them is much shorter than that of the general vehicles, and they can

make immediate reaction.

- There is no entrance and exist on each period of lane divided out in the given Excel table
- In the same time and space, the parallel lanes on a period of road have the same traffic condition.

3 The Discrete Traffic Flow Model

In this section, we build a system with the speed of vehicles, the traffic flow and the traffic density under the condition that the road is unimpeded.

To make the problem simple, in the beginning, we only consider the situation where the two kinds of cars share the lanes on roads (no lane is dedicated to self-driving cars). The difference between the two types of cars is that, when other factors stay the same, the self-driving cars can drive faster, in that the response time of them is much shorter than that of the general vehicles.

Our model start with the equation of the maximum number of vehicles (Road carrying capacity) that a single period of lane could afford in one hour., and according to the Greenshields model, we assume that there was a linear correlation between speed and traffic density.

Then we divide the initial condition into three main kinds, which includes the case under the condition of road unimpeded, where all the cars are in the maximum speed (60 mile per hour); the case under the condition of road blockage, and the case that the road turn crowded from unblocked.

3.1 The basic part

First of all, we need to figure out the max traffic flow when the speed of the vehicle reaches to the maximum value. According to the definition we can get the following equation:

$$\rho = \frac{N}{\Delta x} \tag{1}$$

$$q = \frac{N}{\Delta t} \tag{2}$$

Where: $\rho(x,t)$ is the traffic density, and $q(x,t)$ is the traffic flow. When Δt and Δx are very small, we can assume that:

$$\Delta x = u\Delta t \tag{3}$$

Then through (1), (2) and (3) we can get:

$$q = u\rho \tag{4}$$

Because the traffic flow would reach the maximum value when the vehicles driving with the minimum distance between each two neighboring fronts, and of course, under ideal traffic conditions, standard length and technical indicators, so we can get:

$$q_M = \frac{u}{d} \tag{5}$$

Where: q_M is the max traffic flow, the maximum number of vehicles on the road in unit time. d is the minimum distance between the fronts of two neighboring cars, which is mainly determined by the braking distance, and is commonly calculated by using the following formula in Traffic Engineering:

$$d = d_1 + d_2 + d_3 + d_4 = t_0 u + cu^2 + d_3 + d_4. \tag{6}$$

Where: d_1 is the distancethat the driver travels during the reaction time when needing to brake. d_2 is the braking distance, which can be described by $d_2 = cu^2$, only related to speed with a braking coefficient when under reasonable assumptions. d_3 is the safe distance between two neighboring vehicles. d_4 is the standard length of a vehicle. c is the braking coefficient, when the speed is about 100km/h, it is about 0.013. t_0i s the reaction time, which of the two types of cars are different. Next, from (5) and (6) we can get:

$$q_M = \frac{u}{ut_0 + cu^2 + d_3 + d_4} \tag{7}$$

So,through (4), we can get the max traffic density under which the vehicles can still travel at the max speed:

$$\rho_0 = \frac{q_M}{u_M} = \frac{1}{u_M t_0 + cu_M^2 + d_3 + d_4} \tag{8}$$

Where: u_M is the max speed. ρ_0 is the max traffic density under which the vehicles can still travel at the max speed.

Because the reaction time t_0 of the two types of cars is different, we use superscript m to describe the variates of non-self-driving cars, and superscript c to describe the self-driving, cooperating ones. Taking t_0^m and t_0^c into (8), we can get the max traffic density(under which the vehicles can still travel at the max speed) of the two: ρ_0^m and ρ_0^c Besides, when the traffic gets blocked, we can also figure out the traffic density because the speed of both self-driving cars and non-self-driving cars here are 0, so:

$$\rho_1^m = \rho_1^c = \frac{1}{d_3 + d_4}. \tag{9}$$

Where: ρ_1 is the traffic density when the traffic gets blocked.

According to the Greenshields Model,the speed and traffic density are linear. Since we already have the max traffic density (under which the vehicles can still travel at the max speed), the traffic density when the traffic gets blocked and the speeds corresponding to them, we can get the speed functions of the two types of cars:

$$u^m(\rho) = \begin{cases} u_M, & \rho \le \rho_0^m \\ -\frac{u_M}{\rho_1-\rho_0^m}(\rho-\rho_1), & \rho_0^m \le \rho \le \rho_1 \\ 0, & \rho = \rho_1 \end{cases} \tag{10}$$

$$u^c(\rho) = \begin{cases} u_M, & \rho \le \rho_0^c \\ -\frac{u_M}{\rho_1-\rho_0^c}(\rho-\rho_1), & \rho_0^c \le \rho \le \rho_1 \\ 0, & \rho = \rho_1 \end{cases} \tag{11}$$

3.2 Simulation

Table 1: Values of parameters.

Parameter	c	d_3	d_4	t_0^m	t_0^c
Value	0.013	2 m	5 m	1 sec	0.2 sec

We take reasonable values of these parameters into (6), and then we can get the minimum distance between the fronts of two neighboring cars. After calculating, we get that:

$$d^m = 44, d^c = 22$$

Then take these values into (8), then we get:

$$\rho_0^m = 36,\ \rho_0^c = 72,\ \rho_1{}^m = \rho_1{}^c = 228$$

Using (10) and (11), we figure out the relation between traffic density and the speed of either kind of case.

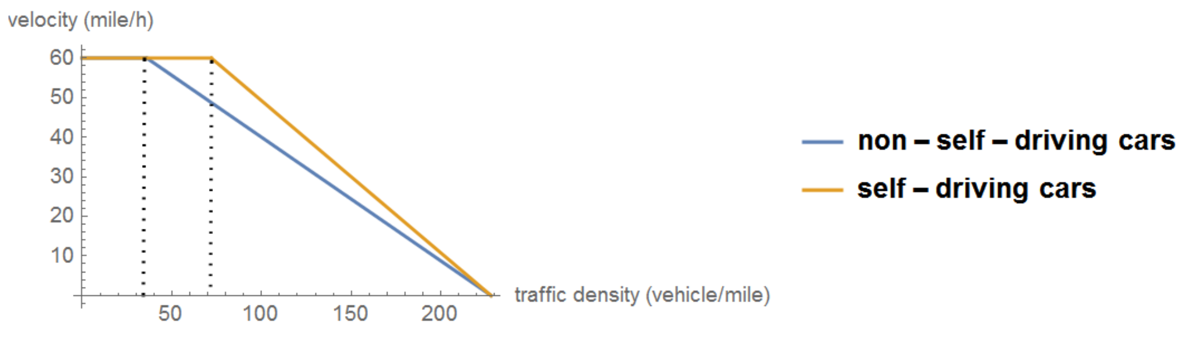

Figure 2: The relationship between traffic density and speed

4 The Continuous PDE Model

4.1 Establishment

Since the Unimpeded Traffic Flow Model is discrete and cannot effectively solve the congestion situation that is the key problem we need to analysis, we established a continuous model based on the LWR model and UTFM (Unimpeded Traffic Flow Model).

- **The LWR Model**

  In the LWR model, considering the vehicles as interacting 'particles', if, as usual, it is assumed that the number of cars is sufficiently large and only the average characteristics of the motion are of interest, the hypothesis of continuity holds, a hydrodynamic description for the distribution of the density of cars in a lane and their average speed is possible, establish a partial differential equation by using two different ways to describe the rate of change of traffic flow over time in a certain section of road. That is:

$$\frac{d}{dt}\int_{x_1}^{x_2}\rho(x,t)\,dx = q(x_1,t)\;-\;q(x_2,t) \tag{12}$$

  And because

$$\frac{d}{dt}\int_{x_1}^{x_2}\rho(x,t)\,dx \;=\; \int_{x_1}^{x_2}\frac{\partial}{\partial t}\rho(x,t)dx \tag{13}$$

$$q(x_1,t)\;-\;q(x_2,t)\;=\;\int_{x_1}^{x_2}-\frac{\partial}{\partial x}q(x,t)dx \tag{14}$$

  Then the LWR equation is established:

$$\frac{\partial}{\partial t}\rho(x,t)+\frac{\partial}{\partial x}q(x,t)=0 \tag{15}$$

  Where:

  x_1 is the starting point of a period of road, and we name it as 'Section α '.

  x_2 is the ending point of the period of the 'Section α'. The values of ρ should be positive and should not exceed the maximum density

- **Mix traffic flow equation**

  Because the speed of the vehicles is only related to the traffic density, using (4), (10) and (11), we can get:

$$\frac{\partial}{\partial t}\rho+\frac{\partial}{\partial x}(\rho u(\rho))=0 \tag{16}$$

  Where: ρ and u(ρ)use corresponding functions and values for different types of cars.

  When the two types of cars mixed together, the model is still applicable. To list and solve the equations, we just need to add up the functions of these two cars, and use the relation between flow and density to describe the total density with flow, then giving the percentage of self-driving cars in total number vehicles different values.

$$\begin{cases} \frac{\partial}{\partial t}\mathrm{P} + \frac{\partial}{\partial x}q^c + \frac{\partial}{\partial x}q^m = 0 \\ \mathrm{P} = \frac{q^c}{u^c} + \frac{q^m}{u^m} \\ \frac{q^c}{q^c+q^m} = p \end{cases} \tag{17}$$

Where:

p is the percentage of the self-driving cars in total number of vehicles.

P is the total density including both two kinds of cars.

Remark:We easily see that the equation (17) is a first-order non-linear partial differential equation, and the PDE has a unique smooth solution with suitable initial and boundary conditions, see for example: Section 3.4 in book [1]

4.2 Initial and boundary condition

To solve the partial differential equations, we have three cases to discuss. Here we only take the non-self-driving cars into consideration, for self-driving cars, using the same solution.

- **Road is unimpeded**

  Because the road is unimpeded, so when t=0, the traffic density function is equal to a constant at all distances, and when x=0, the traffic density function is equal to a constant at all time. And, the vehicles are driving at the max speed when it is unimpeded. Since the two initial boundary values take the same value when both x, t=0, so we can figure out that the traffic density function always equals to a constant that is less than the max traffic density when the speed of the vehicles reaches to the maximum value under the condition that the road is unimpeded.

$$\left.\begin{array}{l} \rho^m(x,0) = a_0 \\ \rho^m(0,t) = a_0 \\ \frac{\partial}{\partial t}\rho^m + u_M\frac{\partial}{\partial x}\rho^m = 0 \\ (a_0 \le {\rho_0}^m) \end{array}\right\} \Rightarrow \rho^m = a_0 \tag{18}$$

- **Road is blocked**

  Because the road is blocked, so when t = 0, the traffic density function is equal to a constant that equals to at all distances, and when x=0, the traffic density function is also equal to at all time. And, the vehicles cannot move at all when it is blocked. Since the two initial boundary values take the same value when both x, t=0,so we can figure out that the traffic density function always equals to under the condition that the road is blocked.

$$\left.\begin{array}{l}\rho^m(x,0)=\rho_1{}^m\\ \rho^m(0,t)=\rho_1{}^m\\ \frac{\partial}{\partial t}\rho^m=0\\ (u=0)\end{array}\right\}\Rightarrow \rho^m=\rho_1{}^m \tag{19}$$

- **The road turn crowded from unblocked**
  Because when t=0, the road is still unblocked, but is going to block, so the first initial boundary value equals to the max traffic density when the speed of the vehicles reaches to the maximum value. When x=0, the traffic density is a function related to time and includes two parts: the traffic density at the very beginning and the density of the adding input of vehicles at the entering junction. The density of the adding input is a function that relates to time and equals to 0 when t=0. Then we get:

$$\left.\begin{array}{l}\rho^m(x,0)=\rho_0{}^m\\ \rho^m(0,t)=\rho_0{}^m+a_1(t)\\ \frac{\partial}{\partial t}\rho^m+\frac{\partial}{\partial x}(u^m\rho^m)=0\\ u^m=u^m(\rho^m)\end{array}\right\}\Rightarrow \rho^m(x,t) \tag{20}$$

  Where: $a_1(t)$is the adding input of vehicles at the entering junction.

4.3 Polishing

Because the functions shown in Figure 1 are unsmooth, so to make it differentiable, we need to use the Polishing Equation, we just polish once, which is:

$$f_{1,h}(x)=\frac{1}{h}\int_{x-\frac{h}{2}}^{x+\frac{h}{2}} f(t)dt \quad (h>0) \tag{21}$$

Assuming that the value of h is 1, then we replace$f_{1,h}(x)$ with $u(\rho)$(including $u^m(\rho)$ and $u^c(\rho)$). After polishing, the former figurer turns smoother, the example point is shown below:

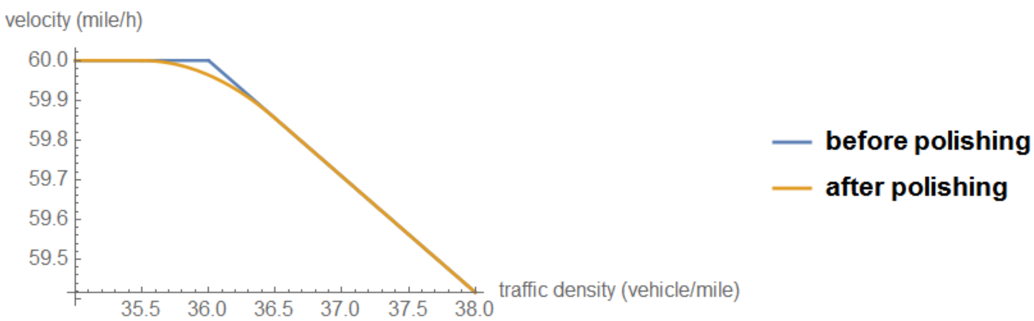

Figure 3: The changes made after polishing

4.4 Numerical solution of PDE

Two extreme conditions

With this polished function, we can then find the numerical solution of (20) with Mathematica. Two extreme conditions are shown as below:

$$p = \frac{q^c}{q^c + q^m} = 0$$

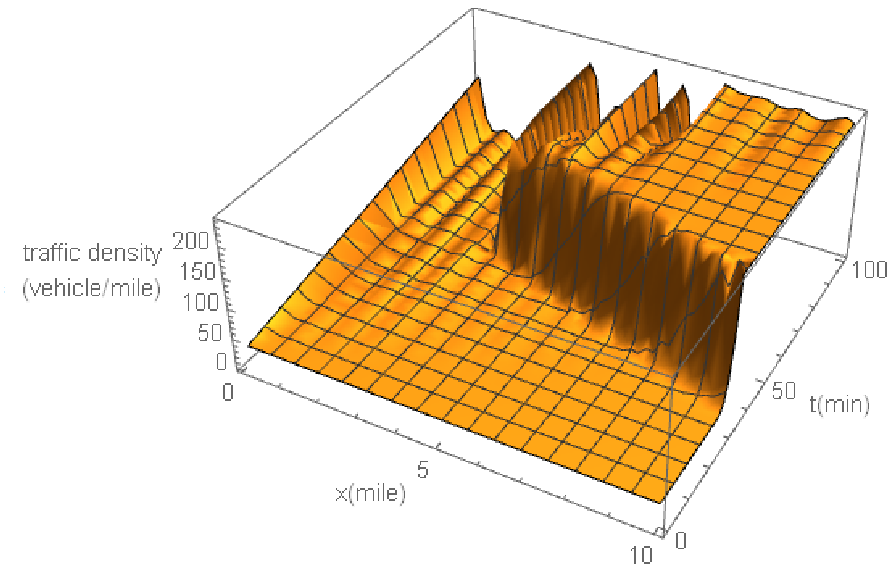

Figure 4: The extreme condition when all the vehicles are non-self-driving

As from the picture, we can see that, when all the vehicles are self-driving, the traffic density tends to be smooth and lower (below 150), which means, the lanes are more difficult to get blocked.

$$p = \frac{q^c}{q^c + q^m} = 1$$

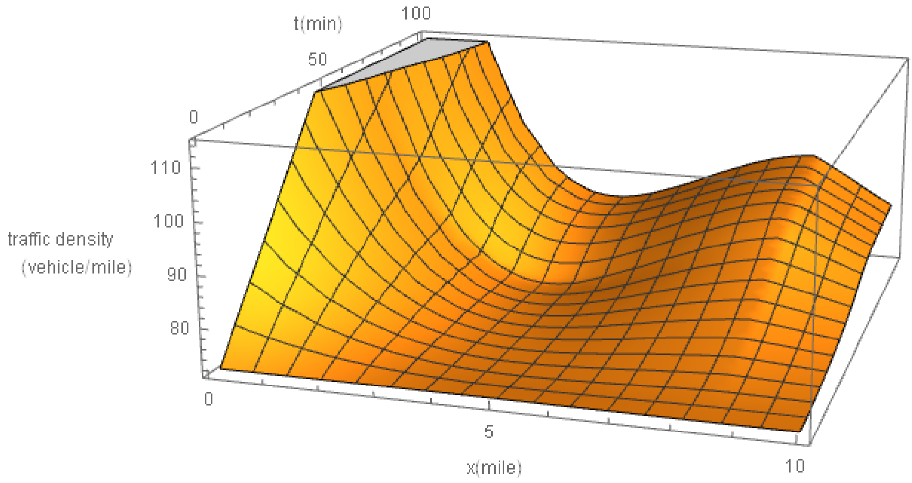

Figure 5: The extreme condition when all the vehicles are non-self-driving

General condition

Step I: Because (20) is related to the unknown parameter , we focus on eleven typical points (the percentage of self-driving cars is 0%, 10%, 20%, 30%, 40% ... 100%). Then according to (20) we can get the corresponding values of (the total density including both two kinds of cars) to each p.
Step II: With the P(x, t) that we figured out in step I, we now can get the total traffic flow in one hour.

$$Q = \int_0^1 \mathrm{P}(x,t)u(\mathrm{P})dt \tag{22}$$

Where: Q is the total flow including both two kinds of cars in one hour.

Table 2: Total traffic flow of different p

P	0%	10%	20%	30%	40%	50%
Q	2115	2176	2364	2576	2915	3364
P	60%	70%	80%	90%	100%	
Q	3674	3845	4025	4139	4229	

According to the above data in Table B, we can get the value of corresponding traffic flow (vehicle per hour), which is shown below:

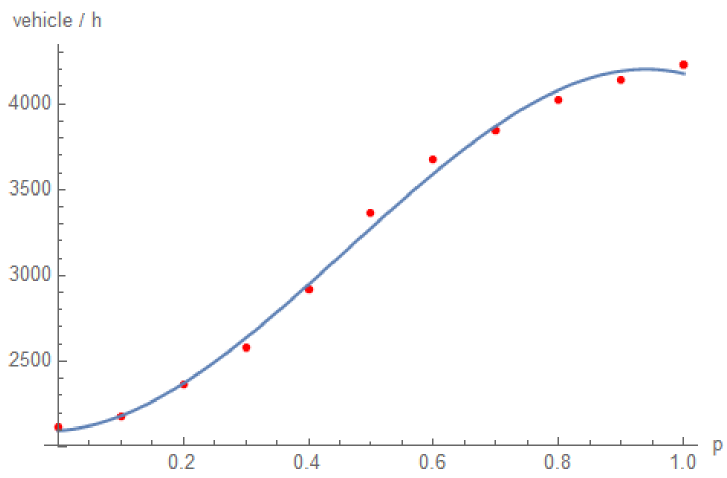

Figure 6: value of corresponding traffic flow

Step III: Using the given data, we compare the average daily traffic counts on one lane in one peak hour with different $P(x,t)$, which correspond to different p , and count the number of sections that is blocked (when the given data of average daily traffic counts on one lane in one peak hour of a section is higher than the total traffic flow Q , we consider it blocked) under different $P(x,t)$.

After collecting statistics and making comparison, we can get a figure:

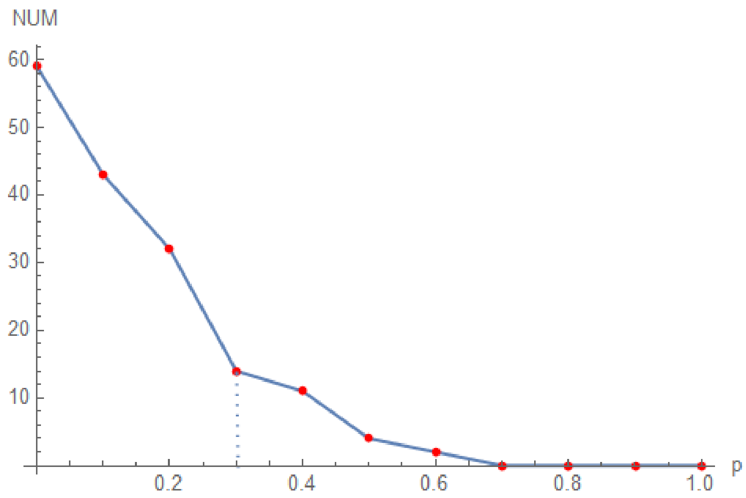

Figure 7: The number of crowded section

The vertical axis represents the number of crowding section among the 224 given sections. According to Figure 6, when all the cars are non-self-driving cars, 58 out of 224 of the total sections will get crowded. With the proportion of self-driving cars increasing, the number of crowding section gets lower, and when reaches about 70%, the number of crowding section gets to 0, which means that, when the proportion of self-driving car reach to 70%, every piece of lane would be unblocked, and 70% is the equilibria.

In the meantime, what we can also learn from Figure 6 is that the number of crowding section reducing much slower when p gets to 30%. That is to say, when gets to 30%, the traffic condition would be improved dramatically. So 30% is the tipping point.

5 Model Extension

5.1 Under what condition would the dedicated lane be valuable?

Since all the self-driving cars can be arranged by cooperating system in the same time, and can contact with each other, so they can speed up and speed down simultaneously. Whats more, we can also keep the cars on the dedicated lane driving in the maximum speed, in that when the traffic flow reach the maximum value, the rest of self-driving cars would be noticed that they have to choose the normal lanes.

However, in the LWR model, we cannot use this character of self-driving cars. In this section, we simply assume that the cars on the lanes are evenly distributed, and the distance between two neighboring head is 17m. Then we can get an equation which is shown below:

$$q_d = \frac{1\ mile}{d_3 + d_4} \cdot u_M \tag{23}$$

where:

- d_3 is the safe distance.
- d_4 is the standard length of vehicles.
- u_M is the maximum speed(60mile/h).
- q_d is the maximum traffic flow on the dedicated lane.

To figure out the critical proportion of self-driving cars, we can use the following equation. (When the percentage of self-driving cars is less than this critical value, all the self-driving cars would choose the dedicated lane; when the proportion goes beyond it, the rest of the self-driving cars need to choose the normal lanes.)

$$p_c = \frac{q_d}{q_d + (n-1)q_n} \tag{24}$$

Where:

- p_c is the critical proportion of self-driving cars.
- q_n is the maximum traffic flow, when all the vehicles on one lane are non-self-driving cars.

- n is the number of parallel lanes towards one same direction in one piece of road.

Because there is only one detected lane, so the number of normal lane is $(n-1)$.

Then in order to find out the relationship between the maximum traffic flow and the proportion of self-driving car, we can use the following equation:

$$q_e = \begin{cases} (n-1)q_0 \cdot \frac{1}{1-p} & p \le p_c \\ q_d + (n-1)q\frac{p-p_c}{1-p_c} & p > p_c \end{cases} \tag{25}$$

Where:

- q_e is the entire traffic flow after the setting of dedicated lane.
- q_0 is the value of traffic flow when the percentage of self-driving cars is 0.(shown in Figure 6)
- q is the corresponding value of traffic flow when the percentage of self-driving cars is different. (shown in Figure 6)

After that we can figure out the corresponding traffic flow of each period of road.

For example:

sample section: a piece of road on the 90th route, whose startMilepost is 3.94 and endMilepost is 5.82

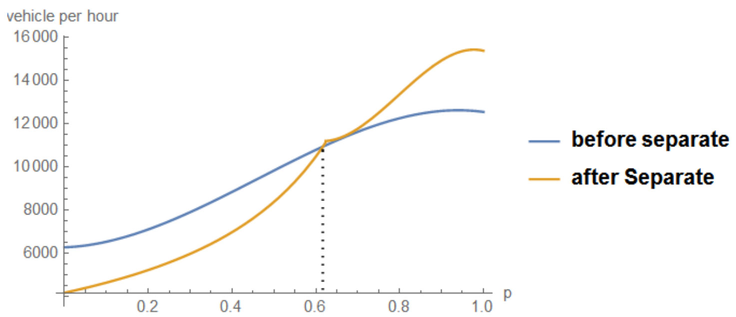

Figure 8: traffic flow condition before and after the setting of a dedicated lane

From this figure we can get that when the proportion reaches about 62%, the yellow traffic flow line exceed the blue one, that is to say, only when the proportion goes beyond 62%, can the dedicated lane be valuable.

5.2 Take junctions, curve lanes and the gesture of changing lanes into consideration

The difference between these traffic conditions and the condition we investigate before is that the PDE equation in LWR model: $\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}(\rho u(\rho)) = 0$ cannot conserve, and instead it turns into $\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}(\rho u(\rho)) = f(x,t)$, where f is related to specific traffic circumstance.

As for the junctions:

$$f(x,t) = \sum_{i=1}^{m} f_i(x_i,t) \tag{26}$$

Where: m is the number of the junctions on the discussed period of road, and as the traffic circumstance at these junctions is different, the corresponding $f(x,t)$ are different either.

As for the curve lanes and the gesture of changing lanes:

Both of these two conditions will cause the speed of cars getting slower, which would also result in no conservation. And under these two conditions $f(x,t)$ is only related to $x: f(x,t) = h(x)$

6 Sensitive analysis

6.1 Change the characteristic of the road

We change the Number of Lanes of the road. And get the change of the critical point Pc where should taking the policy to set a dedicated lane.

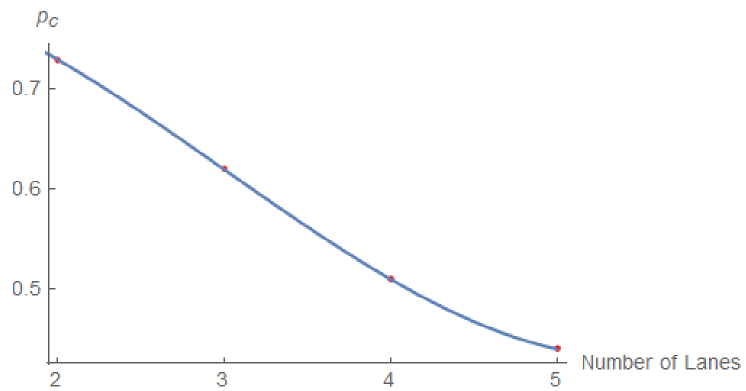

Figure 9: the relationship between number of lanes and the critical point p_c

Where:

- x-axis in Figure 8 represents the change of lanes.
- y-axis in Figure 8 represents the change of the critical point Pc where should taking the policy to set a dedicated lane.

From this figure we can get that the increase of number of lanes causes the decrease of the critical proportion of self-driving cars.

6.2 Change the length of the time

In the formal calculation, we just simulate the road carrying capacity in one hour. To verify the stability of our model, we take a longer period of time and study the traffic flow per hour using different proportion (0.1,0.5,0.9) .

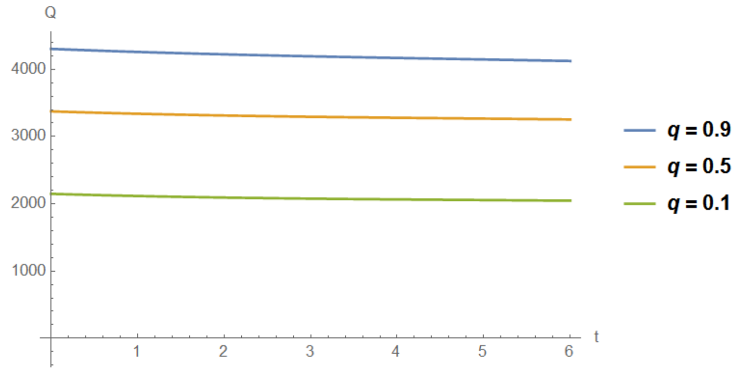

Figure 10: the traffic flow per hour in different percentage

From this figure, we can investigate that the traffic flow Q is stable when time gets longer, regardless of the value of p.

7 Strengths and Weaknesses

7.1 Strengths

- Use the characters of self-driving cars (are able to make reaction immediately; can be arranged as a whole with cooperating system) properly.
- Set appropriate models to describe the interaction between self-driving cars and non-self-driving cars and the cooperation between self-driving cars.
- Two main models which are respectively under the discrete condition and the continuous condition have complementary strength.
- Extend the model, taking the junctions, wave lanes and the gesture of changing lane into consideration, exploring our strategy with practical significance.

7.2 Weaknesses

- The calculation of our model is complex, we need to solve a partial differential equations with parameters.
- Didnt consider the division in the length of different kinds of vehicles.
- Didnt consider the difference between Interstate Route and State Route.
- We only use the data of four routes given in the Excel, and didnt take the effect that the whole road network can bring to solve traffic problem into consideration.

8 Reference

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9 Letter to the governor

A letter to the Governors office

Dear governor,

In response to your question regarding the introduce of self-driving cars, we want to explain our suggestion mainly from three aspects, which includes the specific character of self-driving cars and the improvement they can bring to the current traffic condition; what change would be made as the proportion of self-driving car increases; in what condition would a dedicated lane for self-driving car be valuable.

First of all, the production of self-driving car is a kind of emerging industry. It use the technique of radar detecting or Infrared monitoring, so that it can keep monitoring the surroundings, and make immediate reaction to the accidence (e.g. emergency braking made by the front car), so that the safe distance between a self-driving car and another one as well as the distance between a self-driving car and a non-self-driving car will be shorter than the normal distance between two non-self-driving cars, so that the traffic flow would become smoother, and the traffic condition will become better. In addition, on account of the cooperating control system, the arrangement of self-driving cars could be more flexible.

Secondly, because the self-driving cars havent entered the market broadly yet, and the proportion of them cannot be calculated precisely by now, the application of a detected lane is still in the air, so we investigate the condition when these two kinds of car driving together in the beginning. Furthermore, after making models and calculating, we investigated that when the proportion reach 30%, the problem of traffic congestion in peak hours would be improved dramatically, and when the proportion reach to 70%, all the sections in the listed four routes would become unimpeded. In the end of modeling, we made a prediction that only when the proportion of self-driving cars rises to 62% would the dedicated lane be valuable.

As a conclusion, introduce of self-driving car will generally improve the traffic condition. So we advise you to encourage citizens purchasing self-driving car.

Yours sincerely,

Team # 70112