Peer Effects and Social Networks in Education

By Antoni Calvó-Armengol, Eleonora Patacchini and Yves Zenou

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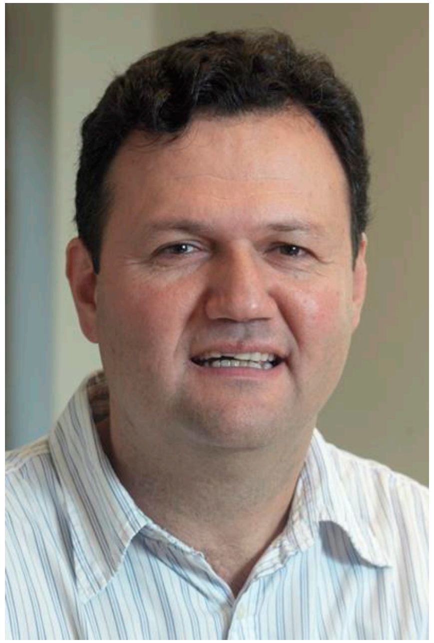
0. Introduction

- About Authors: Antoni Calvó-Armengol, Eleonora Patacchini and Yves Zenou, well known for their work in social networks



0. The Novelty of The Work

- Before this article, there are hardly any studies that have adopted a more structural approach to test a specific peer-effect model in education, and this is exactly what authors do.
  - ✓ First, from a conceptual point of view, they stress the role of the structure of social networks in explaining individual behaviour.
  - ✓ Second, from a more operational point of view, they build a theoretical model of peer effects that envisions group influence as an equilibrium outcome, which aggregates the collection of active dyadic peer influences.
  - ✓ Third, they conduct a direct empirical test of their model on the network structure of peer effects using a detailed dataset on friendship networks, Add Health, with particular attention to the relevant econometric problems.

1. A Network Model of Peer Effects

- The network $N = \{1,\dots,n\}$ is a finite set of agents. We keep track of social connections by a network **g**, where g_{ij} =1 if *i* and *j* are direct friends, and g_{ij} =0, otherwise. Given that friendship is a reciprocal relationship, we set $g_{ij} = g_{ji}$. We also set $g_{ii} = 0$.

- Preferences Denote by y_i^0 the effort of individual *i* absent of any peer influence, and by z_i the peer effort whose returns depend on others' peer efforts. Each agent *i* selects both efforts $y_i^0 \geq 0$ and $z_i \geq 0$, and obtains a payoff $u_i(\mathbf{y}^0, \mathbf{z}; \mathbf{g})$, that depends on the underlying network **g** , in the following way:

1. A Network Model of Peer Effects

$$u_i(\mathbf{y}^0, \mathbf{z}; \mathbf{g}) = \theta_i \, y_i^0 - \frac{1}{2}\left(y_i^0\right)^2 + \mu g_i z_i - \frac{1}{2} z_i^2 + \phi \sum_{j=1}^{n} g_{ij} z_i z_j \qquad (1)$$

- where $\phi > 0, \ \mu > 0$ and $g_i = \sum_{j=1}^{n} g_{ij}$ is the number of direct links of individual *i*. This utility function is additively separable in the idiosyncratic effort component and the peer effect contribution. The component θ_i introduces the *exogenous heterogeneity* that captures the *observable* differences between individuals. Examples of such heterogeneity are agent *i*'s parents' education, neighbourhood where he/she lives, age, sex, race, etc. and also the average characteristics of the individuals directly linked to *i.i.e.* average level of parental education of *i*'s friends, etc. (*contextual effects*).

1. A Network Model of Peer Effects

$$\theta_i(\mathbf{x}) = \sum_{m=1}^{M} \beta_m x_i^m + \frac{1}{g_i} \sum_{m=1}^{M} \sum_{j=1}^{n} \gamma_m g_{ij} x_j^m, \tag{2}$$

- where x_i^m is a set of M variables accounting for observable differences in individual, neighbourhood and school characteristics of individual i, and β_m, γ_m are parameters. The peer-effect component is also *heterogeneous*, and this endogenous heterogeneity reflects the different locations of individuals in the friendship network **g** and the resulting effort levels.

1. A Network Model of Peer Effects

To be more precise, bilateral influences are captured using the following cross derivatives, for $i \neq j$:

$$\frac{\partial^2 u_i(\mathbf{y}^0, \mathbf{z}; \mathbf{g})}{\partial z_i \partial z_j} = \phi g_{ij} \geq 0. \tag{3}$$

When i and j are direct friends, the cross derivative is $\phi > 0$ and reflects *strategic complementarity* in efforts.

1. A Network Model of Peer Effects

$$u_i(\mathbf{y}^0, \mathbf{z}; \mathbf{g}) = \theta_i \, y_i^0 - \frac{1}{2}\left(y_i^0\right)^2 + \mu g_i z_i - \frac{1}{2} z_i^2 + \phi \sum_{j=1}^{n} g_{ij} z_i z_j \tag{1}$$

$$\theta_i(\mathbf{x}) = \sum_{m=1}^{M} \beta_m x_i^m + \frac{1}{g_i} \sum_{m=1}^{M} \sum_{j=1}^{n} \gamma_m g_{ij} \, x_j^m, \tag{2}$$

$$\frac{\partial^2 u_i(\mathbf{y}^0, \mathbf{z}; \mathbf{g})}{\partial z_i \partial z_j} = \phi g_{ij} \geq 0. \tag{3}$$

- The utility function in this model is concave in own decisions, and displays decreasing marginal returns in own effort levels.

➢ The Katz–Bonacich network centrality

The Katz–Bonacich centrality measures the importance of a given node in a network. $0 \leq \phi$ is some non-negative scalar. A factor that decays with the distance discounts the contribution of all these nodes: the value of k−link away nodes is weighted by ϕ^{k-1}. Given a network g and a scalar ϕ, we denote by $\mathbf{b}(\mathbf{g}, \phi)$ the vector whose coordinates correspond to the Katz–Bonacich centralities of all the network nodes.

1. A Network Model of Peer Effects

We associate a matrix **G** =[g_{ij}] The *k*th power $\mathbf{G}^k = \underbrace{\mathbf{G} \cdots \mathbf{G}}_{k\ times}$ keeps track of indirect connections in **g**. More precisely, the coefficient in the *(i , j)* cell of $\mathbf{G}^k$ gives the number of paths of length k in **g** between *i* and *j*. A path between *i* and *j* need not follow the shortest possible route between those agents. For instance, when g_{ij} =1, the sequence *ij* → *ji*→ *ij* constitutes a path of length three in **g** between i and j. Denote by **1** the vector of ones. Then, **G1** is the vector of node connectivities, whereas the coordinates of $\mathbf{G}^k$ **1** give the total number of paths of length k that emanate from the corresponding network node.

1. A Network Model of Peer Effects

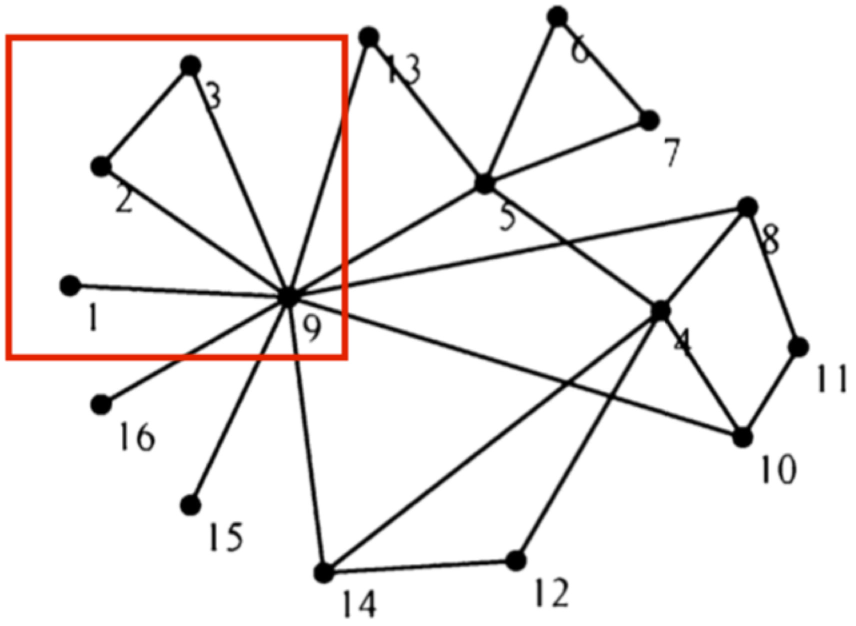


FIGURE 2

Smallest network of adolescents (n = 16)

$$G = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$G^2 = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 3 \end{pmatrix}$$

1. A Network Model of Peer Effects

The vector of Katz–Bonacich centralities is thus:

$$\mathbf{b}(\mathbf{g}, \phi) = \phi\mathbf{G}\mathbf{1} + \phi^2\mathbf{G}^2\mathbf{1} + \phi^3\mathbf{G}^3\mathbf{1} + \cdots = \sum_{k=0}^{+\infty} \phi^k\mathbf{G}^k \cdot (\phi\mathbf{G}\mathbf{1}) .$$

If ϕ is small enough, this infinite sum converges to a finite value, which is $(\mathbf{I} - \phi\mathbf{G})^{-1}$

We can then write the vector of Katz–Bonacich centralities as follows:

$$\mathbf{b}(\mathbf{g}, \phi) = (\mathbf{I} - \phi\mathbf{G})^{-1} \cdot (\phi\mathbf{G}\mathbf{1}) . \tag{4}$$

1. A Network Model of Peer Effects

We now characterize the Nash equilibrium of the game where agents choose their effort levels $y_i^0 \geq 0$ and $z_i \geq 0$ simultaneously. Each individual i

$$u_i(\mathbf{y}^0, \mathbf{z}; \mathbf{g}) = \theta_i\, y_i^0 - \frac{1}{2}\left(y_i^0\right)^2 + \mu g_i z_i - \frac{1}{2} z_i^2 + \phi \sum_{j=1}^{n} g_{ij} z_i z_j \tag{1}$$

maximizes(1) and we obtain the following best reply function for each i =1,...,n:

$$y_i^{0*}(\mathbf{x}) = \theta_i(\mathbf{x}) = \sum_{m=1}^{M} \beta_m x_i^m + \frac{1}{g_i} \sum_{m=1}^{M} \sum_{j=1}^{n} \gamma_m g_{ij}\, x_j^m. \tag{5}$$

$$z_i^*(\mathbf{g}) = \mu g_i + \phi \sum_{j=1}^{n} g_{ij} z_j. \tag{6}$$

1. A Network Model of Peer Effects

The optimal exogenous and endogenous peer efforts are given by (5) and (6), and the individual outcome is the sum of these two different efforts, namely:

$$y_i^* (\mathbf{x}, \mathbf{g}) = \underbrace{y_i^{0*} (\mathbf{x})}_{\text{idiosyncratic}} + \underbrace{z_i^* (\mathbf{g})}_{\text{peer effect}} . \tag{7}$$

In other words, we can decompose additively individual behaviour into an exogenous part and an endogenous peer-effect component that depends on the individual under consideration.

1. A Network Model of Peer Effects

Denote by $\omega(\mathbf{g})$ the largest eigenvalue of the adjacency matrix $\mathbf{G}=[g_{ij}]$ of the network

Proposition 1. Suppose that $\phi\omega(\mathbf{g})<1$. Then, the individual equilibrium outcome is uniquely defined and given by:

$$y_i^* (\mathbf{x}, \mathbf{g}) = \theta_i(\mathbf{x})+ \frac{\mu}{\phi} b_i(\mathbf{g},\phi). \qquad (8)$$

Condition $\phi\omega(g) < 1$ in Proposition 1 requires that the parameter for own-concavity 1 (*i.e.* $\left|\frac{\partial^2 u_i}{\partial z_i^2}\right|$) is high enough to counter the payoff complementarity, measured by $\phi\omega(g)$ in order to prevent the positive feed-back loops triggered by such complementarities to escalate without bound. The scalar ϕ measures the level of positive cross effects (*i.e.* $\left|\frac{\partial^2 u_i}{\partial z_i \partial z_j}\right|$), whereas $\omega(g)$ captures the population-wide pattern of these positive cross effects.

2. Data and Descriptive Evidence

- Friendship network
  - ✓ Pupils were asked to identify their best friends from aschool roster (up to five males and five females).
  - ✓ i.e. a link exists between two friends if at least one of the two individuals has identified the other as his/her best friend.
- Educational achievements
  - ✓ Ranging from D or lower to A, the highest grade (re-coded 1–4).
- Sample
  - ✓ 11,964 pupils distributed over 199 networks.

Descriptive statistics on selected variables

	Mean	Standard deviation	Minimum	Maximum
Female	0·41	0·35	0	1
Black or African American	0·17	0·31	0	1
Other races	0·12	0·15	0	1
Age	15·29	1·85	10	19
Religion practice	3·11	1·01	1	4
Health status	3·01	1·77	0	4
School attendance	3·28	1·86	1	6
Student grade	9·27	3·11	7	12
Motivation in education	2·23	0·88	1	4
Relationship with teachers	0·12	0·34	0	1
Social exclusion	2·26	1·81	1	5
School attachment	2·59	1·76	1	5
Parental care	0·69	0·34	0	1
Household size	3·52	1·71	1	6
Two married parent family	0·41	0·57	0	1
Single parent family	0·23	0·44	0	1
Public assistance	0·12	0·16	0	1
Mother working	0·65	0·47	0	1
Parental education	3·69	2·06	1	5
Parent age	40·12	13·88	33	75
Parent occupation manager	0·11	0·13	0	1
Parent occupation office or sales worker	0·26	0·29	0	1
Parent occupation manual	0·21	0·32	0	1
Neighbourhood quality	2·99	2·02	1	4
Friend attachment	0·49	0·54	0	1
Physical development	3·14	2·55	1	5
Self esteem	3·93	1·33	1	6

2. Data and Descriptive Evidence

REVIEW OF ECONOMIC STUDIES

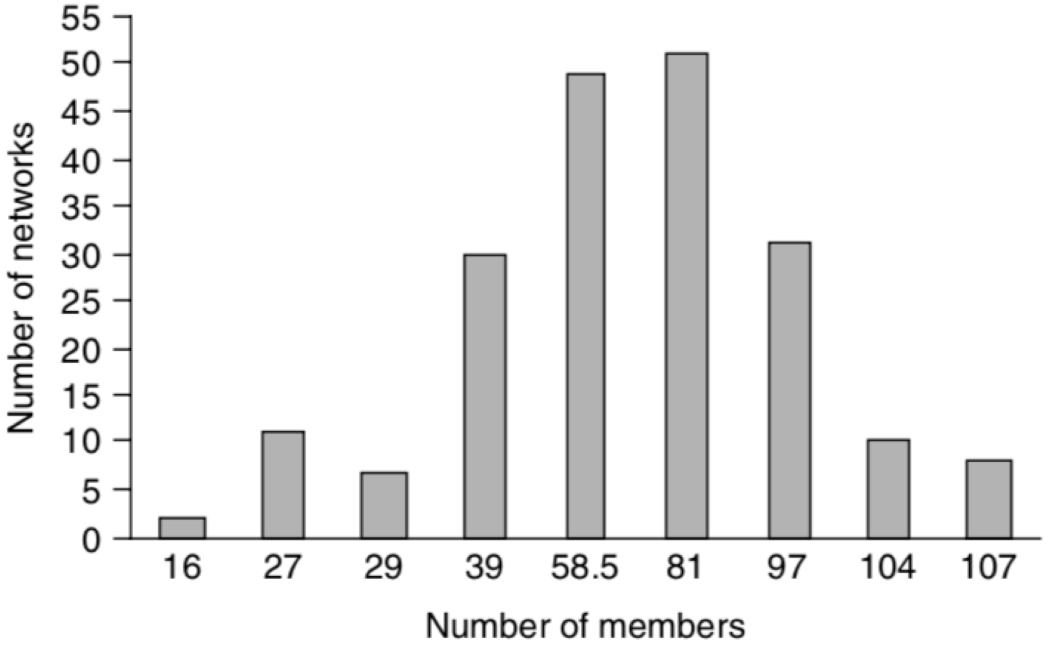


FIGURE 1

The empirical distribution of adolescent networks

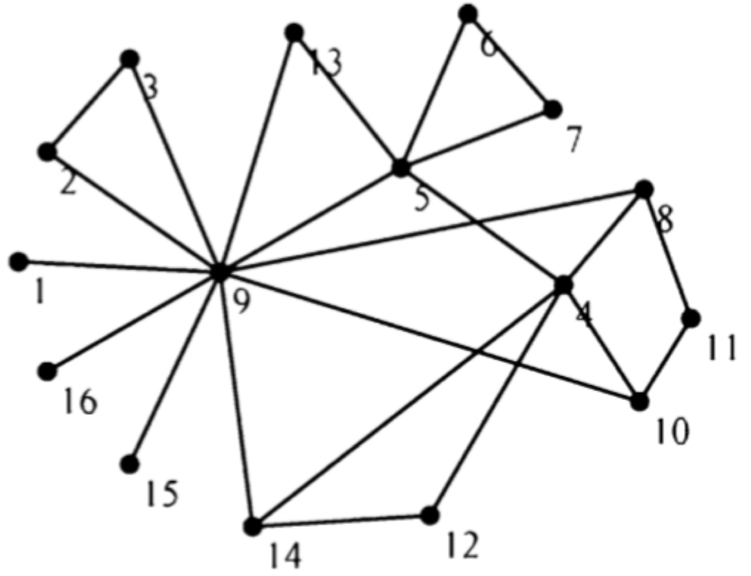


FIGURE 2

Smallest network of adolescents (n = 16)

3. Empirical Strategy and Identification of Peer Effects

3.1 empirical strategy

- K network components
  - ✓ Connectedness: two agents in a network component g_K are either directly linked, or indirectly linked through a sequence of agents in g_K
  - ✓ Maximality: two agents in different network components g_K and g_K cannot be connected through any such sequence
  - ✓ Note that $\sum_K^K n_K = n$.

$$y_{i,\kappa} = \sum_{m=1}^{M} \beta_m x_{i,\kappa}^m + \frac{1}{g_{i,\kappa}} \sum_{m=1}^{M} \sum_{j=1}^{n_\kappa} \gamma_m g_{ij,\kappa} x_{j,\kappa}^m + \eta_\kappa + \varepsilon_{i,\kappa}, \quad (9)$$

$$\varepsilon_{i,\kappa} = \mu g_{i,\kappa} + \phi \sum_{j=1}^{n_\kappa} g_{ij,\kappa} \varepsilon_{j,\kappa} + \upsilon_{i,\kappa}, i = 1, \dots, n; \kappa = 1, \dots, K,$$

3. Empirical Strategy and Identification of Peer Effects

3.1 empirical strategy

$$y_{i,\kappa} = \sum_{m=1}^{M} \beta_m x_{i,\kappa}^m + \frac{1}{g_{i,\kappa}} \sum_{m=1}^{M} \sum_{j=1}^{n_\kappa} \gamma_m g_{ij,\kappa} x_{j,\kappa}^m + \eta_\kappa + \varepsilon_{i,\kappa}, \qquad (9)$$

$$\varepsilon_{i,\kappa} = \mu g_{i,\kappa} + \phi \sum_{j=1}^{n_\kappa} g_{ij,\kappa} \varepsilon_{j,\kappa} + \upsilon_{i,\kappa}, i = 1, ..., n; \kappa = 1, ..., K,$$

- η_κ is an (unobserved) network-specific component (constant over individuals in the same network), which might be correlated with the regressors.
- $\varepsilon_{i,\kappa}$ is the residual of individual i's level of activity in the network g_κ that is not accounted for either by individual heterogeneity and contextual effects or by (unobserved) network-specific components.
- Estimate $\hat{\beta}$, $\hat{\gamma}$, $\hat{\varphi}$ and $\hat{\mu}$ to measure the relative importance of individual characteristics
  - ✓ $\hat{\beta}_1,\ldots\hat{\beta}_m$: contextual effects
  - ✓ $\hat{\gamma}_1,\ldots\hat{\gamma}_m$: individual's best friends
  - ✓ individual Katz–Bonacich centrality index: $\hat{\varphi}$ and $\hat{\mu}$

3. Empirical Strategy and Identification of Peer Effects

3.2 identification of peer effects

- The role of network-fixed effects: the endogenous sorting of individuals into groups
  - ✓ In most cases, individuals sort into groups non-randomly.
    For example, kids whose parents are less educated or worse than average in unmeasured ways would be more likely to sort with low human capital peers.
  - ✓ Subtract the network average from the individual-level variables.

- The role of peer groups with individual level variation: reflection problem
  - ✓ Cannot distinguish if a group member's action is the cause or the effect of peers' influence.
  - ✓ Bramoull´e, Djebbari and Fortin (2009) provide general results on the identification of peer effects through social networks via variations of the linear-in-means model
  - ✓ **G1**=[g_i] gives the total number of one-link away contacts in the network
  - ✓ $\mathbf{G}^2\mathbf{1}$=[g_i^2]gives the total number of two-link away contacts in the network

Proposition 2. *Suppose that $\phi\omega(\mathbf{g}) < 1$ and $\mu \neq 0$. Peer effects are identified if and only if $g_i^{[2]}/g_i \neq g_j^{[2]}/g_j$ for at least two agents i and j.*

3.2 identification of peer effects

- The role of specific controls
  - ✓ find proxies for typically unobserved individual characteristics that may be correlated with our variable of interest.
  - ✓ Therefore, we deal with unobservable individual characteristics correlated with the Katz–Bonacich measure that may cause education outcomes not directly caused by the centrality measure.

- Estimation strategy
  - ✓ First, we estimate our empirical model defined by equation (9) for each network in our dataset.
  - ✓ Second, for each network g_K , we calculate its largest eigenvalue $\omega(g_K)$ and check which network does not satisfy the condition $\varphi_K < 1/\omega(g_K)$.
  - ✓ Estimate model (9) by running a pseudo-panel data estimation (i.e. using both within and between-network variations), thus obtaining an average estimate of φ and μ in our dataset.

4. Empirical Results And Discussion

- The Maximum Likelihood estimation results for the model specification that includes the complete set of controls are reported here.

Model (9): Maximum Likelihood estimation results
Dependent variable: school performance index

	Undirected networks	Directed networks
Number of best friends (μ)	0·0314**	0·0323**
	(0·0149)	(0·0152)
Peer effects (ϕ)	0·5667***	0·5505***
	(0·1433)	(0·1247)
Individual socio-demographic variables	yes	yes
Family background variables	yes	yes
Protective factors	yes	yes
Residential neighbourhood variables	yes	yes
Contextual effects	yes	yes
School-fixed effects	yes	yes
R^2	0.8987	0.8905

- The standard error of μ/φ is calculated using the deltha method. The associated t-test value is equal to 2·11, which denotes statistical significance at the 5% level.

4. Empirical Results And Discussion

- ✓ The estimated μ and φ are both positive and highly statistically significant.
- ✓ The estimated impact of this variable on education outcomes that is predicted by the theory, i.e. μ/φ (equation (8) in Proposition 1) is statistically significant and non-negligible in magnitude.

A one-standard deviation increase in the Katz–Bonacich index translates into roughly 7% of a standard deviation in education outcome.
（This effect is about 17% for parental education.）

4. Empirical Results And Discussion

- The condition $\varphi\omega(g) < 1$ is needed for：
  - ✓ Characterizing the Nash equilibrium in terms of the Katz–Bonacich centrality measure
  - ✓ The existence and uniqueness of the equilibrium as well as for the interiority of the solution.

- Relax the condition $\varphi\omega(g) < 1$ and bound the strategy space by simply acknowledging the fact that students have a time constraint and allocate their time between leisure and school work.
  - ✓ The empirical model is exactly the same with the only difference that we now run the regression on all the 199 networks and not on 181 networks.
  - ✓ When we run such a regression, we obtained that the estimated values of μ and φ are now respectively given by 0·0301 (with a standard error of 0·0140) and 0·5352 (with a standard error of 0·1366).

5. Alternative Formulation

- Limitation and Extentions
  - ✓ Efforts separated into two parts: y_i and z_i
    - ☐ Three alternative network models: only one effort is considered
    - ☐ Equilibrium is equal to weighted Katz-Bonacich centrality index
    - ☐ Not able to distinguish between impact of network location and that of individual characteristics
  - ✓ Positive peer effect
    - ☐ Peer-oriented effort can detract from school outcomes

$y_i^*(\mathbf{x},\mathbf{g}) = \underbrace{y_i^{0*}(\mathbf{x})}_{\text{idiosyncratic}} + \underbrace{z_i^*(\mathbf{g})}_{\text{peer effect}}$	$y_i^*(\mathbf{x},\mathbf{g}) = \underbrace{y_i^{0*}(\mathbf{x})}_{\text{idiosyncratic}} - \underbrace{z_i^*(\mathbf{g})}_{\text{peer effect}}$
$y_i^*(\mathbf{x},\mathbf{g}) = \theta_i(\mathbf{x}) + \frac{\mu}{\phi} b_i(\mathbf{g},\phi)$	$y_i^*(\mathbf{x},\mathbf{g}) = \theta_i(\mathbf{x}) - \frac{\mu}{\phi} b_i(\mathbf{g},\phi)$

- ☐ Difference lies on the sign of μ
  - Average estimate of μ: Positive sign +
  - Separate estimate of μ: 6% of the network have negative μ

6. Alternative Measures of Unit Centrality

Katz-Bonacich centrality	Degree centrality	Closeness centrality	Betweenness centrality
• Counts number of any path stemming from a given node. • A discounting factor as the links increase.	• The number of direct friends.	• Sum of shortest path between *i* and others	• a= number of shortest paths between j and l through I • b= number of shortest paths between j and l
$\mathbf{b}(\mathbf{g}, \phi) = \phi\mathbf{G}\mathbf{1} + \phi^2\mathbf{G}^2\mathbf{1} + \phi^3\mathbf{G}^3\mathbf{1} + \cdots$	$\delta_i(\mathbf{g}_\kappa) = g_i = \sum_{j=1}^{n} g_{ij}$	$c_i(\mathbf{g}_\kappa) = \frac{1}{\sum_j d_{ij,\kappa}},$	$f_i(\mathbf{g}_\kappa) = \sum_{j<l} \frac{a}{b}$

6. Alternative Measures of Unit Centrality

- Results
  - Insignificant
  - In terms of standard deviation: 2.1%

TABLE 4

Explanatory power of different unit centrality measures
Dependent variable: school performance index

	OLS	OLS	OLS
Degree centrality	0·2508* (0·1475)	–	–
Closeness centrality	–	0·2892 (0·2599)	–
Betweenness centrality	–	–	0·0621 (0·0698)

- Explanation
  - Unique Nash equilibrium——linear quadratic utility function——Katz-Bonacich centrality

    It maps topology to equilibrium behavior→behavioral foundation
  - Alternative measures: Parameter-free network indices

    Katz-Bonacich centrality: Depending on **topology** and **peer effect strength (Φ)** .

7. Directed Networks

➢ Assumption: Relationships are reciprocal $g_{ij,\kappa} = g_{ji,\kappa}$

- Truth: 14% relationships are not reciprocal.

➢ Differences

- Adjacency Matrix G turns to asymmetric.
- ω(g) defined as spectral radius rather than largest eigenvalue $\phi\omega(g) < 1$

➢ Results

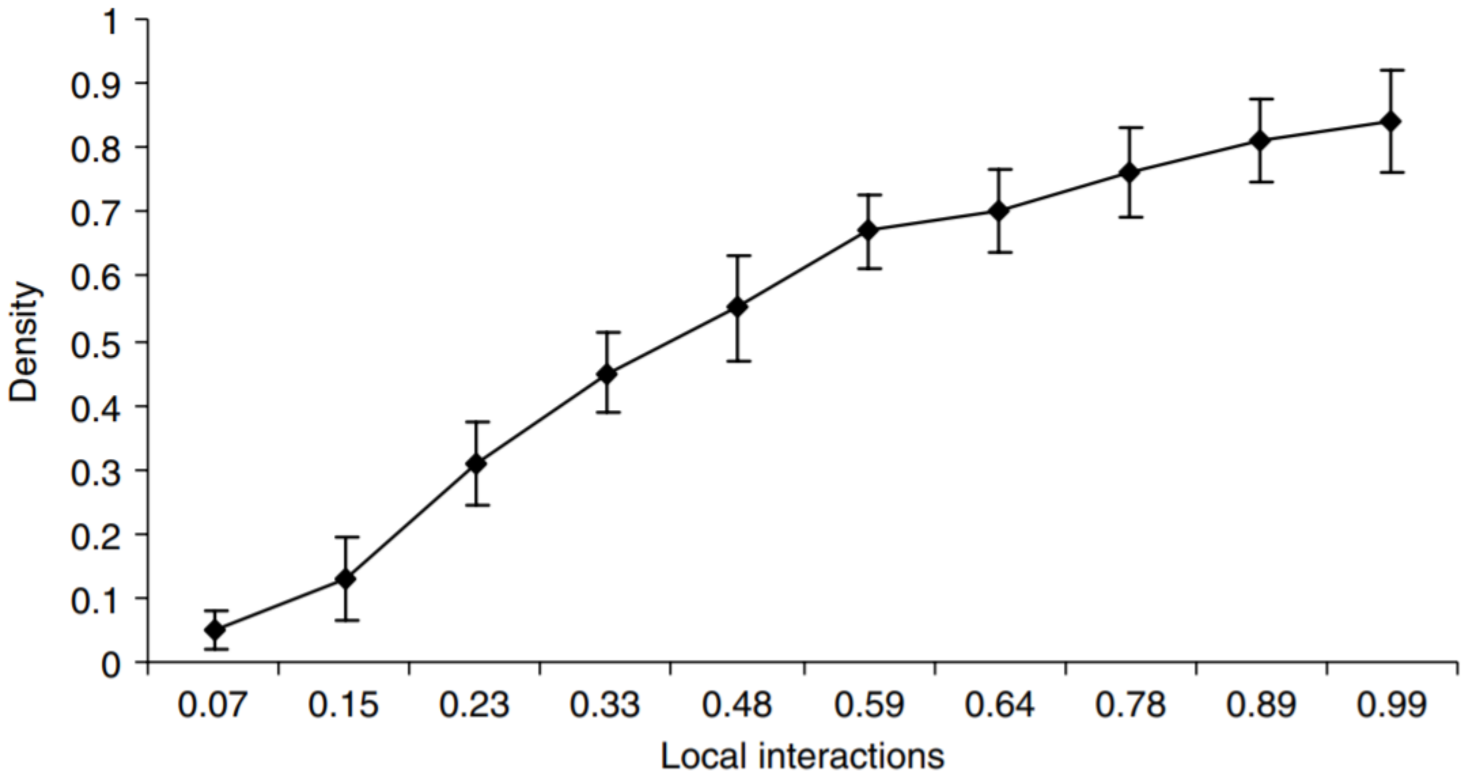
- The estimated effect is still statistically significant
- Slightly lower in magnitude(5.6 vs 7%)

8. Peer Effects and Network Structure

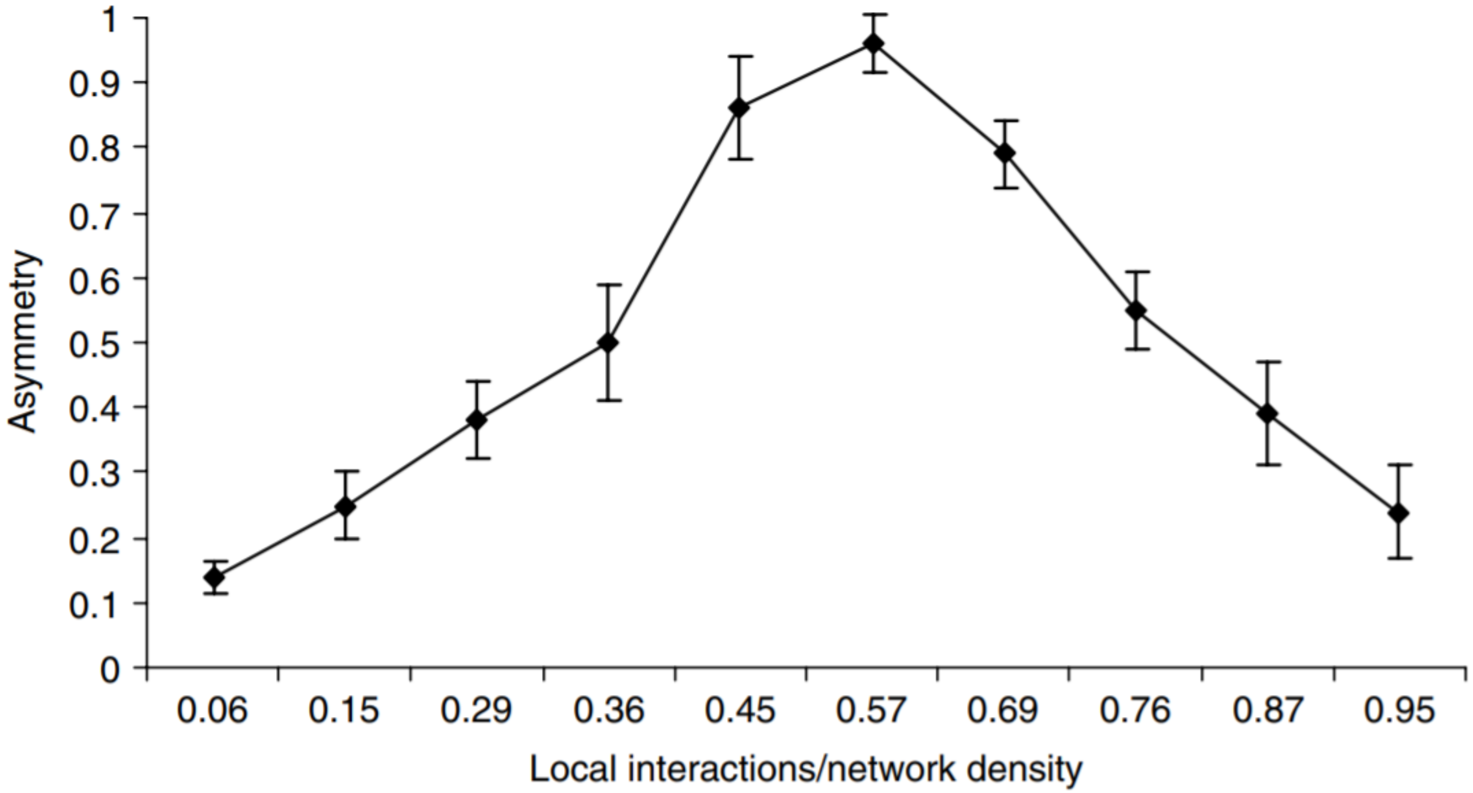
➢Relationship between structural network measures and estimated $\widehat{\phi}_\kappa$

- $\widehat{\phi}_\kappa$ measures the strength of each existing bilateral influence in network.
- All the $\widehat{\phi}_\kappa$ are strictly positive.

◆Density



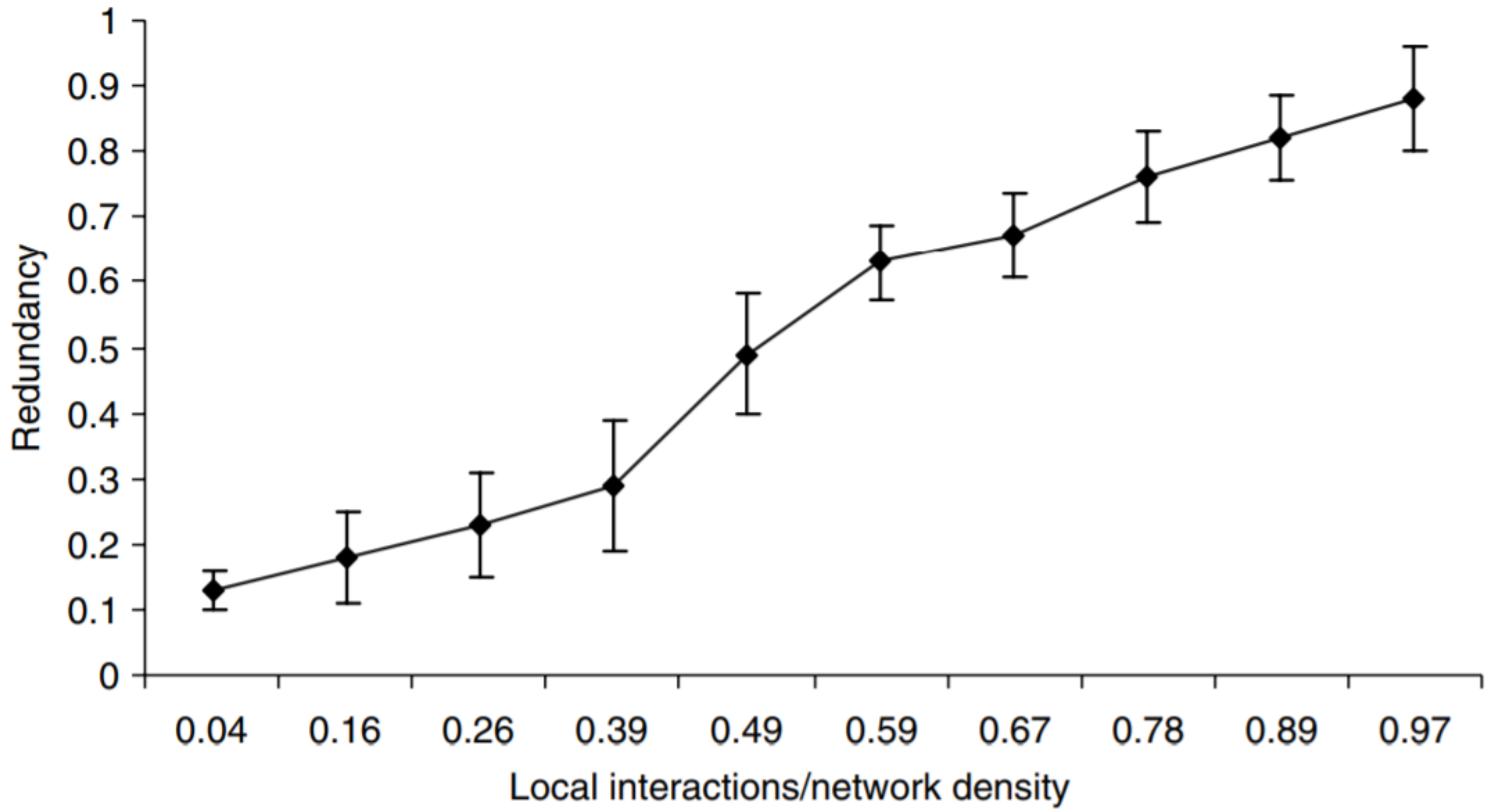
◆Asymmetry: Variance of connectivity

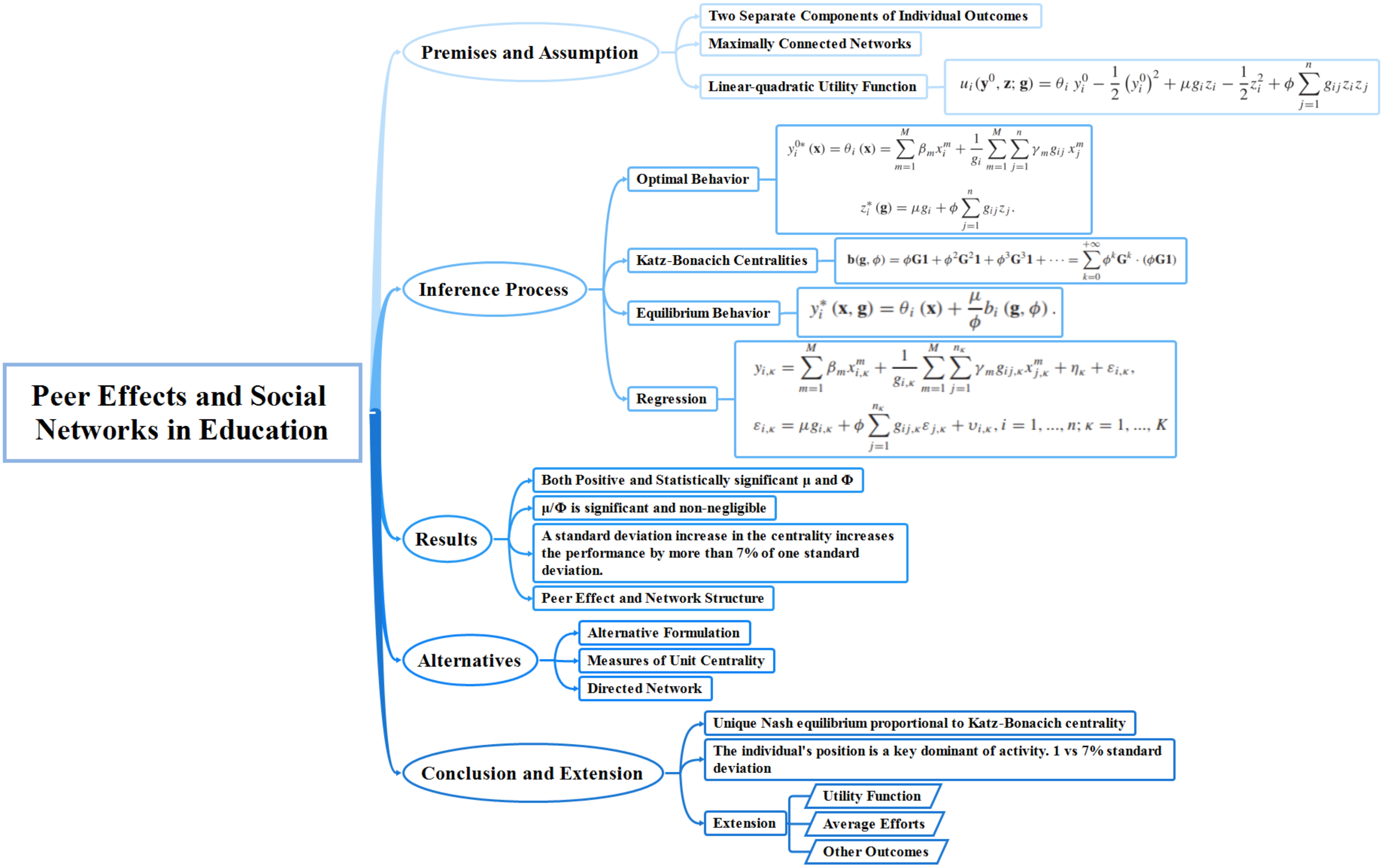


8. Peer Effects and Network Structure

◆Redundancy: Fraction of transitive triads

✓ It measures the probability with which two of i's friends know each other




Peer Effects and Social Networks in Education
Premises and Assumption
Two Separate Components of Individual Outcomes
Maximally Connected Networks
Linear-quadratic Utility Function
$u_i(\mathbf{y}^0, \mathbf{z}; \mathbf{g}) = \theta_i\, y_i^0 - \frac{1}{2}(y_i^0)^2 + \mu g_i z_i - \frac{1}{2} z_i^2 + \phi \sum_{j=1}^{n} g_{ij} z_i z_j$
Inference Process
Optimal Behavior
$y_i^{0*}(\mathbf{x}) = \theta_i(\mathbf{x}) = \sum_{m=1}^{M} \beta_m x_i^m + \frac{1}{g_i} \sum_{m=1}^{M} \sum_{j=1}^{n} \gamma_m g_{ij} x_j^m$
$z_i^*(\mathbf{g}) = \mu g_i + \phi \sum_{j=1}^{n} g_{ij} z_j .$
Katz-Bonacich Centralities
$\mathbf{b}(\mathbf{g}, \phi) = \phi \mathbf{G1} + \phi^2 \mathbf{G}^2 \mathbf{1} + \phi^3 \mathbf{G}^3 \mathbf{1} + \cdots = \sum_{k=0}^{+\infty} \phi^k \mathbf{G}^k \cdot (\phi \mathbf{G1})$
Equilibrium Behavior
$y_i^*(\mathbf{x}, \mathbf{g}) = \theta_i(\mathbf{x}) + \frac{\mu}{\phi} b_i(\mathbf{g}, \phi) .$
Regression
$y_{i,\kappa} = \sum_{m=1}^{M} \beta_m x_{i,\kappa}^m + \frac{1}{g_{i,\kappa}} \sum_{m=1}^{M} \sum_{j=1}^{n_\kappa} \gamma_m g_{ij,\kappa} x_{j,\kappa}^m + \eta_\kappa + \varepsilon_{i,\kappa},$
$\varepsilon_{i,\kappa} = \mu g_{i,\kappa} + \phi \sum_{j=1}^{n_\kappa} g_{ij,\kappa} \varepsilon_{j,\kappa} + \upsilon_{i,\kappa}, i = 1, ..., n; \kappa = 1, ..., K$
Results
Both Positive and Statistically significant μ and Φ
μ/Φ is significant and non-negligible
A standard deviation increase in the centrality increases the performance by more than 7% of one standard deviation.
Peer Effect and Network Structure
Alternatives
Alternative Formulation
Measures of Unit Centrality
Directed Network
Conclusion and Extension
Unique Nash equilibrium proportional to Katz-Bonacich centrality
The individual's position is a key dominant of activity. 1 vs 7% standard deviation
Extension
Utility Function
Average Efforts
Other Outcomes

9. Concluding Remarks

- Conclusion
  - ✓ The peer effects game has a unique Nash equilibrium where each agent strategy is proportional to Katz-Bonacich centrality measure.
  - ✓ The individual's position in a network is a key dominant of his level of activity.
  - ✓ 1 vs 7% standard deviation.
- Extension
  - ✓ Linear-quadratic utility functions.
  - ✓ Consider average effort of peer effect rather than aggregate efforts.
  - ✓ Other outcomes than education could be studied.

Thank you