Appendix: Trade, Migration, and Productivity: A Quantitative Analysis of China

Proposition 1

Given real wages for each region and sector V_i^k*, migration costs between all region-sector pairs* μ_{ni}^{jk} *, land rebates through* δ_{ni}^{jk} *, and a Fréchet distribution* $F_\varepsilon(x)$ *of the heterogeneous preferences, the share of (n, j)-registered workers who migrate to (i, k) is*

$$m_{ni}^{jk} = \frac{\left(\frac{V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}\right)^{\kappa}}{\sum_{k'} \sum_{i'=1}^{N} \left(\frac{V_{i'}^{k'} \delta_{ni'}^{jk'}}{\mu_{ni'}^{jk'}}\right)^{\kappa}}$$

Proof :

The law of large numbers implies that the proportion of workers who migrate to region (i, k) is

$$m_{ni}^{jk} = \Pr\left(\frac{\varepsilon_i^k V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}} \geq \max_{i',k'} \left\{\frac{\varepsilon_{i'}^{k'} V_{i'}^{k'} \delta_{ni'}^{jk'}}{\mu_{ni'}^{jk'}}\right\}\right),$$

where $(i', k') \neq (i, k)$.

Since ε follows a Fréchet distribution with CDF $F_\varepsilon(x) = \Pr(\varepsilon \leq x) = \exp(-x^{-\kappa})$, we have

$$\Pr\left(\frac{\varepsilon_i^k V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}} \le x\right) = \exp\left(-\left(x \,/\, \frac{V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}\right)^{-\kappa}\right) \quad .$$

And distribution of the highest net real income in all other regions is described by

$$\Pr\left(\max_{i',k'}\left\{\frac{\varepsilon_{i'}^{k'} V_{i'}^{k'} \delta_{ni'}^{jk'}}{\mu_{ni'}^{jk'}}\right\} \le x\right) = \prod_{i',k'} \Pr\left(\varepsilon_{i'}^{k'} \le \frac{x\mu_{ni'}^{jk'}}{V_{i'}^{k'} \delta_{ni'}^{jk'}}\right)$$

$$= \exp\left(-\left\{x \,/\, \left[\sum_{i',k'}\left(\frac{V_{i'}^{k'} \delta_{ni'}^{jk'}}{\mu_{ni'}^{jk'}}\right)^{\kappa}\right]^{\frac{1}{\kappa}}\right\}^{-\kappa}\right) .$$

Let $X = \frac{\varepsilon_i^k V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}$, $Y = \max_{i',k'}\left\{\frac{\varepsilon_{i'}^{k'} V_{i'}^{k'} \delta_{ni'}^{jk'}}{\mu_{ni'}^{jk'}}\right\}$,

$S_X = \frac{V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}$, $S_Y = \left[\sum_{i',k'}\left(\frac{V_{i'}^{k'} \delta_{ni'}^{jk'}}{\mu_{ni'}^{jk'}}\right)^{\kappa}\right]^{\frac{1}{\kappa}}$.

By the Law of Total Probability,

$$m_{ni}^{jk} = \int_0^\infty \Pr(X \ge Y | Y = y)\, f_Y(y) dy$$

$$= \int_0^\infty \left[1 - \exp\left(-\left(\frac{y}{S_X}\right)^{-\kappa}\right)\right] \kappa S_Y^{\kappa} y^{-1-\kappa} \exp\left(-\left(\frac{y}{S_Y}\right)^{-\kappa}\right) dy$$

$$= 1 - \int_0^\infty \exp(-(S_X^{\kappa} + S_Y^{\kappa}) y^{-\kappa})\, \kappa S_Y^{\kappa} y^{-1-\kappa} dy \ .$$

With a change of variables $u = y^{-\kappa}$ and therefore $du = -\kappa y^{-1-\kappa} dy$,

$$
\begin{aligned}
m_{ni}^{jk} &= 1 + \int_{\infty}^{0} \exp\left(-u(S_X^{\kappa} + S_Y^{\kappa})\right) S_Y^{\kappa}\, du \\
&= 1 - \frac{S_Y^{\kappa}}{S_X^{\kappa} + S_Y^{\kappa}} \\
&= \frac{S_X^{\kappa}}{S_X^{\kappa} + S_Y^{\kappa}} \\
&= \left(\frac{V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}\right)^{\kappa} \Big/ \sum_k \sum_{i=1}^{N} \left(\frac{V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}\right)^{\kappa},
\end{aligned}
$$

which is the result.

Proposition 2

Given changes in migration and real incomes, the change in aggregate welfare is

$$\widehat{W} = \sum_j \sum_{n=1}^{N} \omega_n^j \hat{V}_n^j \hat{\delta}_{nn}^{jj} \left(\widehat{m}_{nn}^{jj}\right)^{-\frac{1}{\kappa}},$$

where $\omega_n^j \propto V_n^j \delta_{nn}^{jj} \left(m_{nn}^{jj}\right)^{-\frac{1}{\kappa}} \bar{L}_n^j$ *is region n and sector j's initial contribution to welfare.*

Proof :

The probability that a given person's welfare is below x is the probability that no region and sector gives utility above x.

$$\Pr\left(\max_{i,k}\left\{\frac{\varepsilon_i^k V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}\right\} \leq x\right) = \exp\left(-\left\{x \Big/ \left[\sum_{i,k}\left(\frac{V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}\right)^{\kappa}\right]^{\frac{1}{\kappa}}\right\}^{-\kappa}\right)$$

So, the maximized real income of workers from region i and sector k after migration decisions is Frechet with

$$E(U) = \tilde{\gamma}\left[\sum_{i,k}\left(\frac{V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}\right)^{\kappa}\right]^{\frac{1}{\kappa}}.$$

According to Proposition 1 (Notice that $\mu_{nn}^{jj} = 1$),

$$\left(m_{nn}^{jj}\right)^{-\frac{1}{\kappa}} = \left[\left(\frac{V_n^j \delta_{nn}^{jj}}{\mu_{nn}^{jj}}\right)^{\kappa} \Big/ \sum_k \sum_{i=1}^{N}\left(\frac{V_i^k \delta_{ni}^{jk}}{\mu_{ni}^{jk}}\right)^{\kappa}\right]^{-\frac{1}{\kappa}} = E(U) \,/\, \tilde{\gamma} V_n^j \delta_{nn}^{jj},$$

where $\tilde{\gamma} = \Gamma(1-\kappa^{-1})$.

Aggregate average real income (welfare) is therefore

$$W = \sum_j \sum_{n=1}^{N} E(U)\bar{L}_n^j = \sum_j \sum_{n=1}^{N} \tilde{\gamma} V_n^j \delta_{nn}^{jj}\left(m_{nn}^{jj}\right)^{-\frac{1}{\kappa}} \bar{L}_n^j.$$

And the change in aggregate welfare is

$$\widehat{W} = \frac{W'}{W} = \frac{\sum_j \sum_{n=1}^{N} \tilde{\gamma}(V_n^j \hat{V}_n^j)(\delta_{nn}^{jj} \hat{\delta}_{nn}^{jj})\left(m_{nn}^{jj} \widehat{m}_{nn}^{jj}\right)^{-\frac{1}{\kappa}} \bar{L}_n^j}{\sum_j \sum_{n=1}^{N} \tilde{\gamma} V_n^j \delta_{nn}^{jj}\left(m_{nn}^{jj}\right)^{-\frac{1}{\kappa}} \bar{L}_n^j}$$

$$= \sum_j \sum_{n=1}^{N} \omega_n^j \hat{V}_n^j \hat{\delta}_{nn}^{jj}\left(\widehat{m}_{nn}^{jj}\right)^{-\frac{1}{\kappa}},$$

where $\omega_n^j = \frac{\tilde{\gamma} V_n^j \delta_{nn}^{jj}\left(m_{nn}^{jj}\right)^{-\frac{1}{\kappa}} \bar{L}_n^j}{W} \propto V_n^j \delta_{nn}^{jj}\left(m_{nn}^{jj}\right)^{-\frac{1}{\kappa}} \bar{L}_n^j$.

Similarly, the change in GDP is

$$\hat{Y} = \sum_j \sum_{n=1}^{N} \phi_n^j \hat{V}_n^j \hat{L}_n^j ,$$

where $\phi_n^j \propto V_n^j L_n^j$

已知柯布道格拉斯效用函数和预算约束:

$$u_n^j = \varepsilon\left(C_n^{j,ag}\right)^{\alpha\Psi^{ag}}\left(C_n^{j,na}\right)^{\alpha\Psi^{na}}\left(S_n^{j,h}\right)^{1-\alpha}$$

s.t. $P_n^{j,ag}C_n^{j,ag} + P_n^{j,na}C_n^{j,na} + r_n^j S_n^{j,h} \le v_{in}^{kj}$

求证: $D_n^j = \alpha\Psi^j \sum_{k\in\{ag,na\}} v_n^k L_n^k$

解:

$$L = \varepsilon\left(C_n^{j,ag}\right)^{\alpha\Psi^{ag}}\left(C_n^{j,na}\right)^{\alpha\Psi^{na}}\left(S_n^{j,h}\right)^{1-\alpha} + \lambda\left(P_n^{j,ag}C_n^{j,ag} + P_n^{j,na}C_n^{j,na} + r_n^j S_n^{j,h} - v_{in}^{kj}\right)$$

$$\frac{\partial L}{\partial C_n^{j,ag}} = \varepsilon\alpha\Psi^{ag}\left(C_n^{j,ag}\right)^{\alpha\Psi^{ag}-1}\left(C_n^{j,na}\right)^{\alpha\Psi^{na}}\left(S_n^{j,h}\right)^{1-\alpha} + \lambda P_n^{j,ag} = 0$$

$$\frac{\partial L}{\partial C_n^{j,na}} = \varepsilon\left(C_n^{j,ag}\right)^{\alpha\Psi^{ag}}\alpha\Psi^{na}\left(C_n^{j,na}\right)^{\alpha\Psi^{na}-1}\left(S_n^{j,h}\right)^{1-\alpha} + \lambda P_n^{j,na} = 0$$

$$\frac{\partial L}{\partial S_n^{j,h}} = \varepsilon\left(C_n^{j,ag}\right)^{\alpha\Psi^{ag}}\left(C_n^{j,na}\right)^{\alpha\Psi^{na}}(1-\alpha)\left(S_n^{j,h}\right)^{-\alpha} + \lambda r_n^j = 0$$

$$\therefore \frac{\alpha\Psi^{ag}}{C_n^{j,ag}P_n^{j,ag}} = \frac{\alpha\Psi^{na}}{C_n^{j,na}P_n^{j,na}} = \frac{1-\alpha}{S_n^{j,h}r_n^j}$$

又$\because \frac{\partial L}{\partial \lambda} = P_n^{j,ag}C_n^{j,ag} + P_n^{j,na}C_n^{j,na} + r_n^j S_n^{j,h} - v_{in}^{kj} = 0$

则一个从 i,k 到 n,j 的人在商品 n,ag 上的消费为

$$P_n^{j,ag}C_n^{j,ag} = \frac{\alpha\Psi^{ag}}{\alpha\Psi^{ag} + \alpha\Psi^{na} + 1 - \alpha}v_{in}^{kj} = \alpha\Psi^{ag}v_{in}^{kj}$$

所以

$$D_n^{ag} = \sum_j \sum_{k\in\{ag,na\}} \sum_{i=1}^{N} P_n^{j,ag}C_n^{j,ag}L_{in}^{kj} = \alpha\Psi^{ag}\sum_j\sum_k\sum_{i=1}^{N} v_{in}^{kj}L_{in}^{kj}$$

$$= \alpha\Psi^{ag}\sum_j v_n^j L_n^j$$

所以 $D_n^j = \alpha\Psi^j \sum_{k\in\{ag,na\}} v_n^k L_n^k$ 得证