HISTORICAL INTRODUCTION

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Much of the field of mathematical economics is presented in the chapters in this Handbook. Indeed, while the field of "mathematical economics" could be defined, as in the Preface, as one that "includes various applications of mathematical concepts and techniques to economics, particularly economic theory", an alternative approach to defining the field would be to enumerate all its parts. Pragmatically, our definition of the field in this sense is provided by the Table of Contents of the Handbook. We recognize, however, that this definition is not truly complete; considerations of space limitations and priorities have caused the omission of some very active fields of mathematical economics.

A historical perspective will provide the reader with a sharper sense of the background of and interrelationships among the various chapters. We conclude with a list of eleven important developments in mathematical economics over the period since 1961.

This introduction divides the history of mathematical economics into three broad and somewhat overlapping periods: the calculus-based marginalist period (1838–1947), the set-theoretic/linear models period (1948–1960), and the current period of integration (1961–present). These dates are only suggestive. Calculus-based marginalist analysis has never ceased; the set-theoretic/linear models analysis was begun by 1933 and still continues to be significant.

1. The calculus-based marginalist period: 1838–1947

The early period of mathematical economics was one in which economics borrowed methodologies from the physical sciences and related mathematics to develop a formal theory based largely on calculus. By assuming sufficiently smooth functions (e.g., utility and production functions) and maximizing behavior, a reasonably complete theory of the behavior of microeconomic agents and of general equilibrium was developed. The basic mathematical tool was the calculus, particularly the use of total and partial derivatives and Lagrange

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multipliers to characterize maxima. The mathematical foundations of the modern theories of the consumer, the producer, oligopoly, and general equilibrium were developed in this period.

A seminal work, which may be treated as the starting point of mathematical economics,[1] was Cournot (1838). Cournot's contributions may be sorted under two general headings: theory of the firm and the interaction of firms and consumers in single markets. As to the theory of the firm, Cournot's basic hypothesis was that firms choose output levels to maximize profits. He studied and rigorously defined both the cases of perfect competition and of monopoly. As to the interaction of firms and consumers in single markets, Cournot developed both the equating of supply and demand in (single) competitive markets and the problem of oligopoly, where sellers' competition is limited. The "Cournot solution" to oligopoly is still a standard approach, and a suitable generalization plays an important role in the development of game theory. In the Handbook, oligopoly theory is developed in Chapter 11 by Friedman, while game theory is discussed in Chapter 7 by Shubik.

Theory of the firm: Cournot's profit-maximizing hypothesis was extended primarily through the development of the production function concept in the last quarter of the nineteenth century, so that a full theory dealing with demands for inputs as well as supply of output appeared. The development was shared by many authors, such as Walras (1874) [but the production function and the marginal productivity theory did not appear until the third edition (1896)], Wicksteed (1894), Wicksell (1893), and J. B. Clark (1889). Hotelling (1932) gave perhaps the first fully coherent account. In the Handbook, the theory of the firm is surveyed in Chapter 10 by Nadiri.

Theory of the consumer: The development of the theory of consumer demand from maximization of a utility function under a budget constraint was first begun by Gossen (1854), Jevons (1871), and Walras (1874) and elaborated by Marshall (1890). A full deduction of the properties of utility-maximizing demand functions was achieved by Slutsky (1915) and further studied by Hicks and Allen (1934), Hotelling (1935), Georgescu-Roegen (1936), and Wold (1943–44, 1953). The foundations of utility were deepened in several ways: the replacement of cardinal utility by ordinal was due to Fisher (1892) and Pareto (1909); axiomatizations of cardinal utility were due to Frisch (1926, 1932) and Alt (1936); and the revealed preference approach was initiated by Samuelson (1938), and further developed by Houthakker (1950) and Uzawa (1960). Chapter 9 of the Handbook, by Barten and Böhm, surveys the theory of consumer demand.

[1]There are always predecessors. For a study of the prehistory of mathematical economics, see Theocharis (1961).

General equilibrium: The fundamental concept that markets are interrelated and therefore the equilibrium of the economy is characterized by simultaneous equality of supply and demand on all markets is due to Walras (1874). The concept was further developed and expounded by Pareto (1896, 1909). The case that an equilibrium exists was made plausible by showing that the number of equations equalled the number of unknowns [see also Marshall (1890)]. The optimality of the competitive equilibrium was argued by both Walras and Pareto.

Stability of equilibrium: In the case of equilibrium on a single market, the conditions for stability had been discussed by Cournot (1838) and Marshall (1890). The question of stability of general equilibrium was discussed extensively in Walras (1874), though not very rigorously. The first discussions from a rigorous viewpoint appeared in Hicks (1939a), and Samuelson (1941). Important later papers on stability included Arrow and Hurwicz (1958), Hahn (1958), (1962), Arrow, Block and Hurwicz (1959), Uzawa (1961, 1962), and Hahn and Negishi (1962), building not only on Hicks and Samuelson but also on Mosak (1944) and Metzler (1945). In the Handbook, stability is treated in Chapter 16 by Hahn.

Optimal resource allocation: The first systematic calculation of benefits and costs, essentially using the modern concepts of consumers' and producers' surplus, was due to Dupuit (1844). A clear definition of optimality in the presence of many individuals was given by Pareto (1909). The characterization of optimal and sub-optimal states became known as the field of welfare economics; a synthesis of all earlier work was achieved by Hotelling (1938), Bergson (1938), and Hicks (1939b, 1941).

The particular problems of optimization over time were first studied by Ramsey (1928) and, with special reference to exhaustible resources, by Hotelling (1931). The problem of optimization when the range of possible taxes is limited was first studied by Ramsey (1927). None of these papers had much immediate impact but led to very considerable amounts of research in the postwar period.

Generalized bargaining: Edgeworth (1881) first studied the outcomes of an economy in which all kinds of commodity bargains could be made, not merely those possible in a price system. The set of possible outcomes was called the *contract curve*. A generalized version of this concept, known as the *core*, has been further developed in game theory in general and specifically with reference to economic systems; see Chapter 18 of this Handbook, by Hildenbrand.

The culmination of the calculus-based marginalist school, which combined many previous results with newer developments, is found in two classic books which continue to be highly influential: Hicks (1946) and Samuelson (1947).

Each both summarized received theory and developed newer concepts. One new concept in Hicks (1946) was that of temporary equilibrium, which was developed extensively later; in the Handbook, it is the subject of Chapter 19, by Grandmont. Samuelson (1947) incorporated the work on revealed preference and on stability previously referred to.

2. The set-theoretic/linear models period: 1948–1960

The set-theoretic/linear models period was primarily a post-World War II phenomenon in which the earlier calculus basis for mathematical economics was replaced by a set-theoretic basis and by linear models. Using set theory meant greater generality in that the classical assumption of smooth functions could be replaced by more general functions. Using linear models also meant treatment of phenomena that could not be represented by smooth functions e.g. vertices of polyhedral figures. The basic mathematical tools of the set-theoretic approach, including mathematical analysis, convexity, and elements of topology, are summarized in the Handbook in Chapter 1 by Green and Heller.

The new approach had already been set forth in the context of economic growth in an important paper of von Neumann (1937), of which the methodology was even more important than the context. Another work that played an important role in developing the set-theoretic approach was Arrow (1951a). This book was concerned with the axiomatization of social choice theory, but in the process of doing so, it used set-theoretic techniques, which provided a framework for studying the problems of general equilibrium theory. In the Handbook, social choice theory is developed in Chapter 22 by Sen, while mathematical approaches to competitive equilibrium are treated in Part 3, Chapters 15–21.

Two highly influential papers in the development of the theory of general equilibrium were Wald (1933–34) and Arrow and Debreu (1954). Wald (1933–34, 1936) provided the first rigorous analysis of general equilibrium, building upon earlier developments of Zeuthen (1932), Neisser (1932), von Stackelberg (1933), and Schlesinger (1933–34). Arrow and Debreu (1954) and, independently, McKenzie (1954) made extensive use of set-theoretic approaches in formulating the problem of the existence of a competitive equilibrium and proving existence under appropriate conditions.

The existence problem was further analyzed in McKenzie (1955, 1959, 1961), Gale (1955), Nikaidô (1956), and Debreu (1962). An important tool in this analysis was the Kakutani fixed point theorem, in Kakutani (1941) — a generalization of the Brouwer fixed point theorem.

The optimality of competitive equilibrium (welfare economics) was restudied by set-theoretic and convex-set methods by Arrow (1951b), and Debreu (1951, 1954a). The subject of welfare economics is treated in the Handbook in Part 4, Chapters 22–26.

In the theory of the consumer, further axiomatic developments in the utility function, especially in relation to the ordinalist hypothesis were presented in Debreu (1954, 1964) and Rader (1963). This subject is included in the development of consumer theory by Barten and Böhm in Chapter 9. There was also an axiomatization of utility theory for choice among uncertain options. The early paper of Ramsey (1926) was neglected, and the influential contributors were von Neumann and Morgenstern (1947), Marschak (1950), and Herstein and Milnor (1953). Ramsey (1926) had also axiomatized the related concept of subjective probability; this was subsequently developed, largely independently of Ramsey's work, by Savage (1954), building upon the earlier work of de Finetti (1937).

In many respects the applications in this period of set-theoretic concepts to the theory of economic equilibrium culminated in Debreu (1959), a classic book which has been extremely influential and one which has played a role relative to the modern set-theoretic period comparable to that played by Hicks (1946) and Samuelson (1947) relative to the classical calculus-based period. As in the case of the earlier books, Debreu (1959) both summarized the state of the theory and developed extensions, in particular to equilibrium under uncertainty, building upon Arrow (1953). The topic of equilibrium under uncertainty is treated in the Handbook in Chapter 20 by Radner. A book which summarized later developments in applying both set-theoretic and calculus-based concepts to the theory of economic equilibrium was Arrow and Hahn (1971).

This period from 1948 to 1960 was also one that witnessed the development of linear models, with many areas of application and related developments. Essentially systems of linear equations and systems of linear inequalities replaced the use of partial derivatives of the calculus-based marginalist period. The input–output model, a linear model of interindustry relations, had been developed both before and during this period in Leontief (1941, 1966). The related activity analysis model of production was developed in Koopmans, ed. (1951), Morgenstern, ed. (1954), Koopmans (1957), and, in the Soviet Union, by Kantorovich (1942, 1959). The von Neumann multisector growth model (1937) was the subject of attention in this period, in particular, in Kemeny, Morgenstern, and Thompson (1956), and Gale (1956). This model has played an important role in both general equilibrium theory and growth theory.

Linear programming was developed in this period, stemming from the work of Dantzig (1949, 1951, 1963), although there had been earlier results on systems of linear inequalities. This approach culminated in Dorfman, Samuelson and Solow (1958) and Gale (1960). These books treated not only linear programming, but also linear models of general equilibrium and linear growth models. Of fundamental importance was the development during this period of a related model of capital accumulation in Malinvaud (1953). Dorfman, Samuelson and Solow (1958) presented the initial formulation of the turnpike theorem, which was later proved in Radner (1961), Morishima (1961, 1964), McKenzie (1963), and Nikaidô (1964). In the Handbook, linear programming is treated in Chapter 2 by

Intriligator, and the theory of growth and turnpike theorems is treated in Chapter 26 by McKenzie.

Game theory was also in the process of development in this period, based, in part, on the analysis of linear models. Its origins dated back to von Neumann (1928) but the fundamental developments appeared in von Neumann and Morgenstern (1947) and Nash (1950). The developments of game theory over this period are summarized in Luce and Raiffa (1957). Game theory is treated in the Handbook in Chapter 7 by Shubik.

3. The current period of integration: 1961–present

The current period is one of integration, in which modern mathematical economics combines elements of calculus, set theory, and linear models. It is also a period in which mathematical ideas have been extended to virtually all areas of economics. There are many topics in mathematical economics under development in the current period, which has been and continues to be an extremely fruitful one for mathematical economics. This section presents eleven important topics under development in this period from 1961 to the late 1970's.

(1) Uncertainty and information:[2] Included are the theory of risk aversion, as developed in Pratt (1964) and in Arrow (1970); equilibrium under uncertainty, in Diamond (1967) and Radner (1968); microeconomic applications, in McCall (1971); insurance, in Borch (1968); search behavior, in Rothschild (1974) and Lucas and Prescott (1974); and market signalling, in Spence (1974). In the Handbook the economics of uncertainty is treated in Chapter 6 by Lippman and McCall, information is treated in Chapter 23 by Arrow, the microeconomic theory of investment under uncertainty is treated in Chapter 13 by Merton, and equilibrium under uncertainty is treated in Chapter 20 by Radner.

(2) Global analysis: Mathematical methods which combine calculus and topology are used to study properties of economic equilibria and their variation with respect to changes in the underlying economy. Debreu (1970) pioneered with a study of the conditions under which there are only a finite set of equilibria. In the Handbook, the mathematics of global analysis is the subject of Chapter 8, by Smale, while the applications to economics are surveyed in Chapter 17 by Dierker.

[2]While the analysis of uncertainty is based on the theory of probability and statistics, this analysis should not be confused with econometrics, which refers rather to the inductive study of empirical data by statistical methods in order to estimate economics relationships and to test economic hypotheses, as opposed to the deductive study of formal theories in mathematical economics.

(3) Duality theory: This is an approach to many aspects of economic theory that combines set-theoretic and calculus techniques. Important works in this area include Hotelling (1932, 1935), Roy (1947), McKenzie (1956–57), Shephard (1953, 1970), Samuelson (1953–54), Uzawa (1964a), Chipman (1966), Diewert (1974), and Fuss and McFadden, eds. (1978). In the Handbook, Chapter 12, by Diewert, discusses duality approaches to microeconomic theory.

(4) Aggregate demand functions: The theory of the consumer shows that demand functions of utility-maximizing individuals must satisfy some restrictive conditions. To what extent, if any, are these or similar conditions necessarily true of aggregate demand functions? Sonnenschein (1973) first gave arguments suggesting that aggregated demand functions are not restricted by the condition that the individual demand functions arise from utility maximization. Subsequent important papers are those of Mantel (1974) and Debreu (1974). This topic is discussed in the Handbook in Chapter 14 by Shafer and Sonnenschein.

(5) Core of economy and markets with a continuum of traders: The intuitive concept of a "large" number of traders basic to the hypothesis of perfect competition has been formalized in recent work as either a limit as the number of traders goes to infinity or as a continuum of traders. In large economies, as Edgeworth (1881) had already stated, the core (or contract curve) tends to coincide with the set of competitive equilibria. This theory combines elements of game theory, general equilibrium theory, and measure theory. This analysis was developed in Shubik (1959), Scarf (1962), Debreu and Scarf (1962), Aumann (1964, 1966), Vind (1964, 1965), and in Hildenbrand (1968, 1970a, 1970b). The core of an economy is treated in the Handbook in Chapter 18 by Hildenbrand. Measure theory is the subject of Chapter 5 by Kirman.

(6) Temporary equilibrium: The concept of temporary equilibrium was introduced by Hicks (1939). In such an equilibrium trade takes place sequentially, with each agent forecasting his or her future endowments on the basis of current and past states of the economy. The equilibrium can involve all prices moving fast enough to clear all markets or, alternatively, allow for quantity rationing. This subject is treated in the Handbook in Chapter 19 by Grandmont.

(7) Computation of equilibrium prices: This is a particular case of the computation of fixed points of mappings in which the fixed point is interpreted as an equilibrium price vector, the implied allocation being a feasible one that clears all markets. The major work in this area is Scarf (1967, 1973). This topic is covered in the Handbook in Chapter 21 by Scarf.

(8) Social choice theory: Social choice theory is concerned with the aggregation of individual preferences into social choices. The modern literature on this

subject stems largely from Arrow (1951a), a book that developed the framework for analyzing this problem and that introduced the possibility and impossibility theorems. According to the possibility theorem majority rule satisfies certain axioms of social choice when there are only two alternatives for the society. According to the impossibility theorem, if there are three or more alternatives for the society then no system of aggregation, including majority rule, can satisfy the axioms of social choice. Much of the literature on this subject up to the 1960's is treated in Sen (1970). Social choice is discussed in the Handbook in Chapter 22 by Sen.

(9) Optimal taxation: Early work in this area included that of Ramsey (1927) and Hotelling (1938), while important recent articles include Boiteux (1956), Mirrlees (1971), and Diamond and Mirrlees (1971). This topic is treated in the Handbook in Chapter 24 by Mirrlees, dealing with optimal taxation as an element of normative second-best theory. Chapter 25, by Sheshinski, discusses positive second-best theory.

(10) Optimal growth theory: This area has been developed in Samuelson and Solow (1956), Samuelson (1965), Uzawa (1964b), Koopmans (1965, 1967), Cass (1965, 1966), von Weizsäcker (1965), Gale (1967), Shell, ed. (1967), and Cass and Shell, eds. (1976). In fact, the problem was initially formulated as the problem of optimal savings in an article that was decades ahead of its time, that of Ramsey (1928). The problem was then addressed using more modern tools of analysis and combining this theory with that of multisector growth models in the 1960's. Growth theory and turnpike theorems are treated in the Handbook in Chapter 26 by McKenzie. The mathematical basis of optimal growth theory includes the theory of dynamical systems, as discussed in Chapter 3 by Varian, and control theory, as discussed in Chapter 4 by Kendrick.

(11) Organization theory: This area includes team theory, decentralization, the problem of incentives, and planning. Important earlier works in this area include Simon (1957), Hurwicz (1960), and Marschak and Radner (1972). This topic is represented in the Handbook in Chapter 27 by Marschak, Chapter 28 by Hurwicz, and Chapter 29 by Heal.

By way of summary, eleven important topics in mathematical economics since 1961 have been:

1. *Uncertainty and information* (Chapters 6, 13, 20, 23)
2. *Global analysis* (Chapters 8, 17)
3. *Duality theory* (Chapter 12)
4. *Aggregate demand functions* (Chapter 14)

5. *Core of an economy and markets with a continuum of traders* (Chapters 5, 7, 18)
6. *Temporary equilibrium* (Chapter 19)
7. *Computation of equilibrium prices* (Chapter 21)
8. *Social choice theory* (Chapter 22)
9. *Optimal taxation* (Chapters 24, 25)
10. *Optimal growth theory* (Chapters 3, 4, 26)
11. *Organization theory* (Chapters 27, 28, 29)

References

Alt, F. (1936), "Über die Messbarkeit des Nutzens", Zeitschrift für Nationalökonomie, 7:161–169. Translated as: "On the measurability of utility", in: J. S. Chipman, L. Hurwicz, M. K. Richter and H. F. Sonnenschein, eds., Preferences, utility and demand. New York: Harcourt Brace Jovanovich.

Arrow, K. J. (1951a), Social choice and individual values. New York: Wiley. In 1963, 2nd ed.

Arrow, K. J. (1951b), "An extension of the basic theorems of welfare economics", in: J. Neyman, ed., Proceedings of the 2nd Berkeley symposium on mathematical statistics. Berkeley, CA: University of California Press.

Arrow, K. J. (1953), "Le rôle des valeurs boursières pour la répartition la meilleure des risques", Econometrie, 41–48. In 1964 translated as: "The role of securities in the optimal allocation of risk-bearing", Review of Economic Studies, 31:91–96.

Arrow, K. J. (1970), Essays in the theory of risk-bearing. Amsterdam: North-Holland.

Arrow, K. J., D. Block and L. Hurwicz (1959), "On the stability of the competitive equilibrium, II", Econometrica, 27:82–109.

Arrow, K. J. and G. Debreu (1954), "Existence of equilibrium for a competitive economy", Econometrica, 22:265–290.

Arrow, K. J. and F. Hahn (1971), General competitive analysis. San Francisco, CA: Holden-Day.

Arrow, K. J. and L. Hurwicz (1958), "On the stability of the competitive equilibrium, I", Econometrica, 26:522–552.

Aumann, R. J. (1964), "Markets with a continuum of traders", Econometrica, 32:39–50.

Aumann, R. J. (1966), "Existence of competitive equilibria in markets with a continuum of traders", Econometrica, 34:1–17.

Bergson, A. (1938), "A reformulation of certain aspects of welfare economics", Quarterly Journal of Economics, 53:310–334.

Boiteux, M. (1956), "Sur la gestion des monopoles publics astreints à l'équilibre budgétaire", Econometrica, 24:22–40.

Borch, K. H. (1968), The economics of uncertainty. Princeton, NJ: Princeton University Press.

Cass, D. (1965), "Optimum growth in an aggregative model of capital accumulation", Review of Economic Studies, 32:233–240.

Cass, D. (1966), "Optimum growth in an aggregative model of capital accumulation: A turnpike theorem", Econometrica, 34:833–850.

Cass, D. and K. Shell, eds. (1976), The Hamiltonian approach to dynamic economics. New York: Academic Press.

Chipman, J. S. (1966), "A survey of the theory of international trade: Part 3, The modern theory", Econometrica, 34:18–76.

Chipman, J. S., L. Hurwicz, M. K. Richter and H. F. Sonnenschein, eds. (1971), Preferences, utility and demand. New York: Harcourt Brace Jovanovich.

Clark, J. B. (1889), "The possibility of a scientific law of wages", Publications of the American Economic Association, 4:37–69.

Cournot, A. (1838), Recherches sur les principes mathématiques de la théorie des richesses. In 1929 translated as: Researches into the mathematical principles of the theory of wealth. New York: Macmillan.
Dantzig, G. B. (1949), "Programming of interdependent activities, II: Mathematical model", Econometrica, 17:200–211.
Dantzig, G. B. (1951), "Maximization of a linear function of variables subject to linear inequalities", in: T. C. Koopmans, eds., Activity analysis of production and allocation. New York: Wiley.
Dantzig, G. B. (1963), Linear programming and extensions. Princeton, NJ: Princeton University Press.
Debreu, G. (1951), "The coefficient of resource utilization", Econometrica, 19: 273–292.
Debreu, G. (1954a), "Valuation equilibrium and Pareto optimum", Proceedings of the National Academy of Sciences, 40:588–592.
Debreu, G. (1954b), "Representation of a preference ordering by a numerical function", in: R. M. Thrall, C. H. Coombs and R. L. Davis, eds., Decision processes. New York: Wiley.
Debreu, G. (1959), Theory of value. New York: Wiley.
Debreu, G. (1962), "New concepts and techniques for equilibrium analysis", International Economic Review, 3:257–273.
Debreu, G. (1964), "Continuity properties of Paretian utility", International Economic Review, 5:285–293.
Debreu, G. (1970), "Economies with a finite set of equilibria", Econometrica, 38:387–392.
Debreu, G. (1974), "Excess demand functions", Journal of Mathematical Economics, 1:15–23.
Debreu, G. and H. Scarf (1963), "A limit theorem on the core of an economy", International Economic Review, 4:235–246.
de Finetti, B. (1937), "La prevision: Ses lois logiques, ses sources subjectives", Annales de l'Institut Henri Poincaré, 7:1–68.
Diamond, P. A. (1967), "The role of the stock market in a general equilibrium model with technological uncertainty", American Economic Review, 57:759–776.
Diamond, P. A. and J. Mirrlees (1971), "Optimal taxation and public production – I, II", American Economic Review, 61:8–27, 261–278.
Diewert, E. (1974), "Applications of duality theory", in: M. D. Intriligator and D. A. Kendrick, eds., Frontiers of quantitative economics, Vol. II. Amsterdam: North-Holland.
Dorfman, R., P. A. Samuelson and R. M. Solow (1958), Linear programming and economic analysis. New York: McGraw-Hill.
Dupuit, J. (1844), "De la mesure de l'utilité des travaux publics", Annales des Ponts et Chausées, 2nd Series, 8:332–375. In 1952 translated as: "On the measurement of the utility of public works", International Economic Papers, 2:83–110.
Edgeworth, F. Y. (1881), Mathematical psychics. London: Routledge & Kegan Paul.
Fisher, I. (1892), "Mathematical investigations in the theory of value and prices", in: Transactions of the Connecticut Academy of Arts and Sciences, Vol. 9. New Haven, CT: Connecticut Academy of Art and Sciences.
Frisch, R. (1926), "Sur un problème d'économie pure". Norsk Matematisk Forenings Skrifter, 16:1–40. In 1957 reprinted in: Metroeconomica, 9:79–111. Translated as: "On a problem in pure economics", in: J. S. Chipman, L. Hurwicz, M. K. Richter and H. F. Sonnenschein, eds., Preferences, utility and demand. New York: Harcourt Brace Jovanovich.
Frisch, R. (1932), New methods of measuring marginal utility. Tübingen: Mohr.
Fuss, M. and D. McFadden, eds. (1978), Production economics: A dual approach to theory and applications. Amsterdam: North-Holland.
Gale, D. (1955), "The law of supply and demand", Mathematica Scandinavica, 3:155–169.
Gale, D. (1956), "The closed linear model of production", in: H. W. Kuhn and A. W. Tucker, eds., Linear inequalities and related systems. Princeton, NJ: Princeton University Press.
Gale, D. (1960), The theory of linear economic models. New York: McGraw-Hill.
Gale, D. (1967), "On optimal development in a multi-sector economy", Review of Economic Studies, 34:1–18.
Georgescu-Roegen, N. (1936), "The pure theory of consumer's behavior", Quarterly Journal of Economics, 50:545–593.
Gossen, H. H. (1854), Entwickelung der Gesetze des menschlichen Verkehrs und der daraus fliessenden Regeln für menschliches Handeln. Braunschweig: Fr. Vieweg and Sohn.

Hahn, F. H. (1958), "Gross substitutes and the dynamic stability of general equilibrium", Econometrica, 26:169–170.
Hahn, F. H. (1962), "On the stability of pure exchange equilibrium", International Economic Review, 3:206–213.
Hahn, F. H. and T. Negishi (1962), "A theorem on non-tatônnement stability", Econometrica, 30:463–469.
Herstein, I. N. and J. Milnor (1953), "An axiomatic approach to measurable utility", Econometrica, 21:291–297.
Hicks, J. R. (1939a), Value and capital. New York: Oxford University Press.
Hicks, J. R. (1939b), "The foundations of welfare economics", Economic Journal, 49:696–712.
Hicks, J. R. (1941), "The rehabilitation of consumers' surplus", Review of Economic Studies, 8:108–116.
Hicks, J. R. (1946), Value and capital, 2nd ed. New York: Oxford University Press.
Hicks, J. R. and R. G. D. Allen (1934), "A reconsideration of the theory of value", Economica, 1:52–76.
Hildenbrand, W. (1968), "The core of an economy with a measure space of economic agents", Review of Economic Studies, 35:443–452.
Hildenbrand, W. (1970a), "Existence of equilibria for economies with production and a measure space of consumers", Econometrica, 38:608–623.
Hildenbrand, W. (1970b), "On economies with many agents", Journal of Economic Theory, 2:161–188.
Hotelling, H. (1931), "The economics of exhaustible resources", Journal of Political Economy, 39:137–175.
Hotelling, H. (1932), "Edgeworth's taxation paradox and the nature of demand and supply functions", Journal of Political Economy, 40:577–616.
Hotelling, H. (1935), "Demand functions with limited budgets", Econometrica, 3:66–78.
Hotelling, H. (1938), "The general welfare in relation to problems of taxation and of railway and utility rates", Econometrica, 6:242–269.
Houthakker, H. (1950), "Revealed preference and the utility function", Economica, 17:159–174.
Hurwicz, L. (1960), "Optimality and informational efficiency in resource allocation processes", in: K. J. Arrow, S. Karlin and P. Suppes, eds., Mathematical methods in the social sciences, 1959. Stanford, CA: Stanford University Press.
Jevons, W. S. (1871), The theory of political economy. London and New York: Macmillan. In 1965, 5th ed. New York: A. M. Kelley.
Kakutani, S. (1941), "A generalization of Brouwer's fixed point theorem", Duke Mathematical Journal, 8:451–459.
Kantorovich, L. V. (1942), "On the translocation of masses" (in Russian), Dokl. Akad. Nauk U.S.S.R., 37:199–201.
Kantorovich, L. V. (1959), Economic calculation of optimal utilization of resources (in Russian). Moscow: Publishing House of the Academy of Sciences of the U.S.S.R. Translated as: The best uses of economic resources. Oxford: Pergamon Press.
Kemeny, J. G., O. Morgenstern and G. L. Thompson (1956), "A generalization of the von Neumann model of an expanding economy", Econometrica, 24:115–135.
Koopmans, T. C., ed. (1951), Activity analysis of production and allocation. New York: Wiley.
Koopmans, T. C. (1957), Three essays on the state of economic science. New York: McGraw-Hill.
Koopmans, T. C. (1965), "On the concept of optimal economic growth", in: The econometric approach to development planning. Amsterdam: North-Holland.
Koopmans, T. C. (1967), "Objectives, constraints, and outcomes in optimal growth models", Econometrica 35:1–15.
Leontief, W. W. (1941), The structure of the American economy, 1919–1939. New York: Oxford University Press. In 1951, 2nd ed.
Leontief, W. W. (1966), Input–output economics. New York: Oxford University Press.
Lucas, R., Jr. and E. Prescott (1974), "Equilibrium search and unemployment", Journal of Economic Theory, 7:188–209.
Luce, R. D. and H. Raiffa (1957), Games and decisions. New York: Wiley.
Malinvaud, E. (1953), "Capital accumulation and the efficient allocation of resources", Econometrica, 21:233–268.

Mantel, R. (1974), "On the characterization of aggregate excess demand", Journal of Economic Theory, 7:348–353.
Marschak, J. (1950), "Rational behavior, uncertain prospects, and measurable utility", Econometrica, 18:111–141.
Marschak, J. and R. Radner (1972), Economic theory of teams. New Haven, CT: Yale University Press.
Marshall, A. (1890), Principles of economics. London and New York: Macmillan.
McCall, J. (1971), "Probabilistic microeconomics", The Bell Journal of Economics and Management Science, 2:403–433.
McKenzie, L. (1954), "On equilibrium in Graham's model of world trade and other competitive systems", Econometrica, 22:147–161.
McKenzie, L. (1955), "Competitive equilibrium with dependent consumer preferences", in: H. A. Antosiewiez, ed., Proceedings of the 2nd symposium on linear programming. Washington, DC: National Bureau of Standards.
McKenzie, L. (1956–1957), "Demand theory without a utility index", Review of Economic Studies, 24:185–189.
McKenzie, L. (1959), "On the existence of general equilibrium for a competitive market", Econometrica, 27:54–71.
McKenzie, L. (1961), "On the existence of general equilibrium: Some corrections", Econometrica, 29:247–248.
McKenzie, L. (1963), "Turnpike theorems for a generalized Leontief model", Econometrica, 31:165–180.
Metzler, L. (1945), "The stability of multiple markets: The Hicks conditions", Econometrica, 13:277–292.
Mirrlees, J. (1971), "An exploration in the theory of optimal income taxation", Review of Economic Studies, 38:175–208.
Morgenstern, O., ed. (1954), Economic activity analysis. New York: Wiley.
Morishima, M. (1961), "Proof of a turnpike theorem: The 'no joint production' case", Review of Economic Studies, 28:89–97.
Morishima, M. (1964), Equilibrium, stability, and growth. New York: Oxford University Press.
Mosak, J. L. (1944), General equilibrium theory in international trade. Bloomington, IN: Principia.
Nash, J. F., Jr. (1950), "Equilibrium in *n*-person games", Proceedings of the National Academy of Sciences, 36:48–49.
Neisser, H. (1932), "Lohnhöhe und Beschäftigungsgrad im Marktgleichgewicht", Weltwirtschaftliches Archiv, 36:415–455.
Nikaidô, H. (1956), "On the classical multilateral exchange problem", Metroeconomica, 8:135–145.
Nikaidô, H. (1964), "Persistence of continual growth near the von Neumann ray: A strong version of the Radner turnpike theorem", Econometrica, 32:151–162.
Pareto, V. (1896), Cours d'économie politique. Lausanne: Rouge.
Pareto, V. (1909), Manuel d'économie politique. Paris: Giard.
Pratt, J. W. (1964), "Risk aversion in the small and in the large", Econometrica, 32:122–136.
Rader, J. T. (1963), "The existence of a utility function to represent preferences", Review of Economic Studies, 30:229–232.
Radner, R. (1961), "Paths of economic growth that are optimal with regard only to final states: A turnpike theorem", Review of Economic Studies, 28:98–104.
Radner, R. (1968), "Competitive equilibrium under uncertainty", Econometrica, 36:31–58.
Ramsey, F. P. (1926), "Truth and probability". In 1931 published in: F. P. Ramsey, The foundations of mathematics and other logical essays. London: K. Paul, Trench, Trubner, & Co.
Ramsey, F. P. (1927), "A contribution to the theory of taxation", Economic Journal, 37:47–61.
Ramsey, F. P. (1928), "A mathematical theory of saving", Economic Journal, 38:543–559.
Rothschild, M. (1974), "Searching for the lowest price when the distribution of prices is unknown", Journal of Political Economy, 82:689–711.
Roy, R. (1942), De l'utilité. Paris: Hermann.
Roy, R. (1947), "La distribution du revenu entre les divers biens", Econometrica, 15:205–225.
Samuelson, P. A. (1938), "A note on the pure theory of consumer's behavior", Economica N.S., 5:61–71.

Samuelson, P. A. (1941), "The stability of equilibrium: Comparative statics and dynamics", Econometrica 9:97–120.
Samuelson, P. A. (1947), Foundations of economic analysis. Cambridge: Harvard University Press.
Samuelson, P. A. (1953–54), "Prices of factors and goods in general equilibrium", Review of Economic Studies, 21:1–20.
Samuelson, P. A. (1965), "A catenary turnpike theorem involving consumption and the golden rule", American Economic Review, 55:486–496.
Samuelson, P. A. and R. M. Solow (1956), "A complete capital model involving heterogeneous capital goods", Quarterly Journal of Economics, 70:537–562.
Savage, L. J. (1954), The foundation of statistics. New York: Wiley.
Scarf, H. E. (1962), "An analysis of markets with a large number of participants", in: M. Maschler, ed., Recent advances in game theory. Princeton, NJ: Princeton University Press.
Scarf, H. E. (1967), "On the computation of equilibrium prices", in: Ten economic studies in the tradition of Irving Fisher. New York: Wiley.
Scarf, H. E. (1973), The computation of economic equilibria. New Haven, CT: Yale University Press.
Schlesinger, K. (1933–34), "Über die Produktiongleichungen der ökonomischen Wertlehre", Ergebnisse eines Mathematischen Kolloquiums, 6: 10–11.
Sen, A. K. (1970), Collective choice and social welfare. San Francisco, CA: Holden-Day.
Shell, K., ed. (1967), Essays on the theory of optimal economic growth. Cambridge, MA: MIT Press.
Shepard, R. W. (1953), Cost and production functions. Princeton, NJ: Princeton University Press.
Shepard, R. W. (1970), Theory of cost and production functions. Princeton, NJ: Princeton University Press.
Shubik, M. (1959), "Edgeworth market games", in: A. W. Tucker and R. D. Luce, eds., Contributions to the theory of games, IV. Princeton, NJ: Princeton University Press.
Simon, H. (1957), Models of man. New York: Wiley.
Slutsky, E. (1915), "Sulla teoria del bilancio del consumatore", Giornale degli Economisti, 51:19–23. In 1952 translated as: "On the theory of the budget of the consumer", in: G. Stigler and K. Boulding, eds., Readings in price theory. Homewood, IL: Richard D. Irwin.
Sonnenschein, H. (1965), "A study of the relation between transitive preference and the structure of choice", Econometrica, 33:642–735.
Sonnenschein, H. (1973), "Do Walras' identity and continuity characterize the class of community excess demand functions?", Journal of Economic Theory, 6:345–354.
Spence, A. M. (1974), Market signaling. Cambridge, MA: Harvard University Press.
Theocharis, R. (1961), Early developments in mathematical economics. London: Macmillan.
Uzawa, H. (1960), "Preference and rational choice in the theory of consumption", in: K. J. Arrow, S. Karlin and P. Suppes, eds., Mathematical methods in the social sciences, 1959. Stanford, CA: Stanford University Press.
Uzawa, H. (1961), "The stability of dynamic processes", Econometrica, 29:617–631.
Uzawa, H. (1962), "On the stability of Edgeworth's barter process", International Economic Review, 3:218–232.
Uzawa, H. (1964a), "Duality principles in the theory of cost and production", International Economic Review, 5:216–220.
Uzawa, H. (1964b), "Optimal growth in a two-sector model of capital accumulation", Review of Economic Studies, 31:1–24.
Vind, K. (1964), "Edgeworth allocations in an exchange economy with many traders", International Economic Review, 5:165–177.
Vind, K. (1965), "A theorem on the core of an economy", Review of Economic Studies, 32:47–48.
von Neumann, J. (1928), "Zur Theorie der Gesellschaftsspiele", Mathematische Annalen, 100:295–320. In 1959 translated in: A. W. Tucker and R. D. Luce, eds., Contributions to the theory of games. Princeton: Princeton University Press.
von Neumann, J. (1937), "Über ein ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpunktsatzes", Ergebnisse eines Mathematischen Kolloquiums, 8:73–83. In 1945 translated as: "A model of general economic equilibrium", Review of Economic Studies, 13:1–9.

von Neumann, J. and O. Morgenstern (1947), The theory of games and economic behavior, 2nd ed. Princeton, NJ: Princeton University Press.

von Stackelberg, H. (1933), "Zwei kritische Bemerkungen zur Preistheorie Gustav Cassels", Zeitschrift für Nationalökonomie, 4:456–472.

von Weizsäcker, C. C. (1965), "Existence of optimal programs of accumulation for an infinite time horizon", Review of Economic Studies, 32:85–104.

Wald, A. (1933–34), "Über die eindeutige positive Lösbarkeit der neuen Produktionsgleichungen", Ergebnisse eines Mathematischen Kolloquiums, 6:12–20.

Wald, A. (1934–35), "Über die Produktionsgleichungen der ökonomische Wertlehre", Ergebnisse eines Mathematischen Kolloquiums, 7:1–6.

Wald, A. (1936), "Über einige Gleichungssysteme der mathematischen Ökonomie", Zeitschrift für Nationalökonomie, 7:637–670. In 1951 translated as: "On some systems of equations of mathematical economics", Econometrica, 19:368–403.

Walras, L. (1874), Elements d'economie politique pure. Lausanne: L. Corbaz. In 1954 translated by William Jaffé as: Elements of pure economics. Homewood, IL: Richard D. Irwin.

Wicksell, K. (1893), Über Wert, Kapital und Rente nach den neuen nationalökonomischen Theorien. In 1954 translated as: Value, capital and rent. London: Allen & Unwin.

Wicksteed, P. H. (1894), An essay on the co-ordination of the laws of production. London: Macmillan. In 1932 reprinted. London: London School of Economics and Political Science.

Wold, H. (1943–44), "A synthesis of pure demand analysis", Scandinavisk Aktuarietidskrift, 26:85–118 and 200–263, 27:69–120.

Wold, H., in association with L. Jureen (1953), Demand analysis. New York: Wiley.

Zeuthen, F. (1932), "Das Prinzip der Knappheit, technische Kombination und ökonomische Qualität", Zeitschrift für Nationalökonomie, 4:1–24.