MONOPOLISTIC COMPETITION: BEYOND THE CONSTANT ELASTICITY OF SUBSTITUTION

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PART ONE

1

Drawbacks of CES model and Introduction

Drawbacks of CES model

- Preferences lack flexibility.
- Prices and markups are not affected by firm entry and market size.
- There is no scale effect, that is, the size of firms is independent of the number of consumers.
- Firms'price and size are independent from the geographical distribution of demand.

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Introduction

- preferences: additively separable across varieties
- relative love for variety, the elasticity of the marginal utility $r_u(x) = \frac{1}{\sigma(x)}$
- The market outcome depends on how the relative love of variety varies with the consumption level

Three extensions

- Multisector economy: a differentiated good X , a homogeneous good Y
- Heterogeneous firms: variable cost functions V (q,θ)
- Nonadditive preferences such as quadratic and translog (homogeneous firms and constant marginal costs)

PART TWO

2

The model

Demand

- $$\max_{x_i \geq 0} \mathcal{U} \equiv \int_0^N u(x_i)\, di \quad \text{such that} \quad \int_0^N p_i x_i \, di = E,$$

- $$p_i(x_i) = u'(x_i)/\lambda,$$

- $$\lambda = \frac{\int_0^N x_i u'(x_i)\, di}{E}.$$

-

RLV

$$r_u(x) \equiv -\frac{xu''(x)}{u'(x)} > 0.$$

$$\frac{1}{r_u(x)} = -\frac{1}{\mathcal{E}_p(x)} = -\mathcal{E}_x(p).$$

Supply

$$\max_{q_i \geq 0} \pi(q_i) = R(q_i) - C(q_i) \equiv \frac{u'(q_i/L)}{\lambda} q_i - V(q_i) - F.$$

$$u'(q_i/L) + (q_i/L)u''(q_i/L) = \lambda V'(q_i),$$

$$[2 - r_{u'}(q_i/L)]r_u(q_i/L) - [1 - r_u(q_i/L)]r_C(q_i) > 0 \quad \text{for all } q_i \geq 0,$$

FEE

$$\mathcal{E}_R(\bar{x}) = \mathcal{E}_C(L\bar{x}),$$

PART THREE 3

The Market Outcome

3.1 Existence and Uniqueness of a FEE

1. Existence

$$\varepsilon_R(\bar{x}) = \varepsilon_C(L\bar{x})$$

$$0 \leq \varepsilon_C(0) < \varepsilon_R(0) < \infty, \quad \varepsilon_R(\infty) < \varepsilon_C(\infty)$$

2. Uniqueness

$$pq - V(q) = \frac{V'(q)}{1 - r_u(q/L)} q - V(q) = F$$

$$[2 - r_u'(q_i/L)]r_u(q_i/L) - [1 - r_u(q_i/L)]r_C(q_i) > 0$$

3.1 Existence and Uniqueness of a FEE

- PROPOSITION 1

  If the above two conditions hold, then a unique FEE exists.

  Furthermore, the FEE satisfies the conditions:

$$
\begin{cases}
\varepsilon_R(\bar{x}) = \varepsilon_C(L\bar{x}) \\
\bar{M} = r_u(\bar{x}) \\
\bar{q} = L\bar{x} \\
\bar{N} = EL/C(\bar{q})
\end{cases}
$$

3.2 Market Size

L increases ($L_1 \rightarrow L_2$)

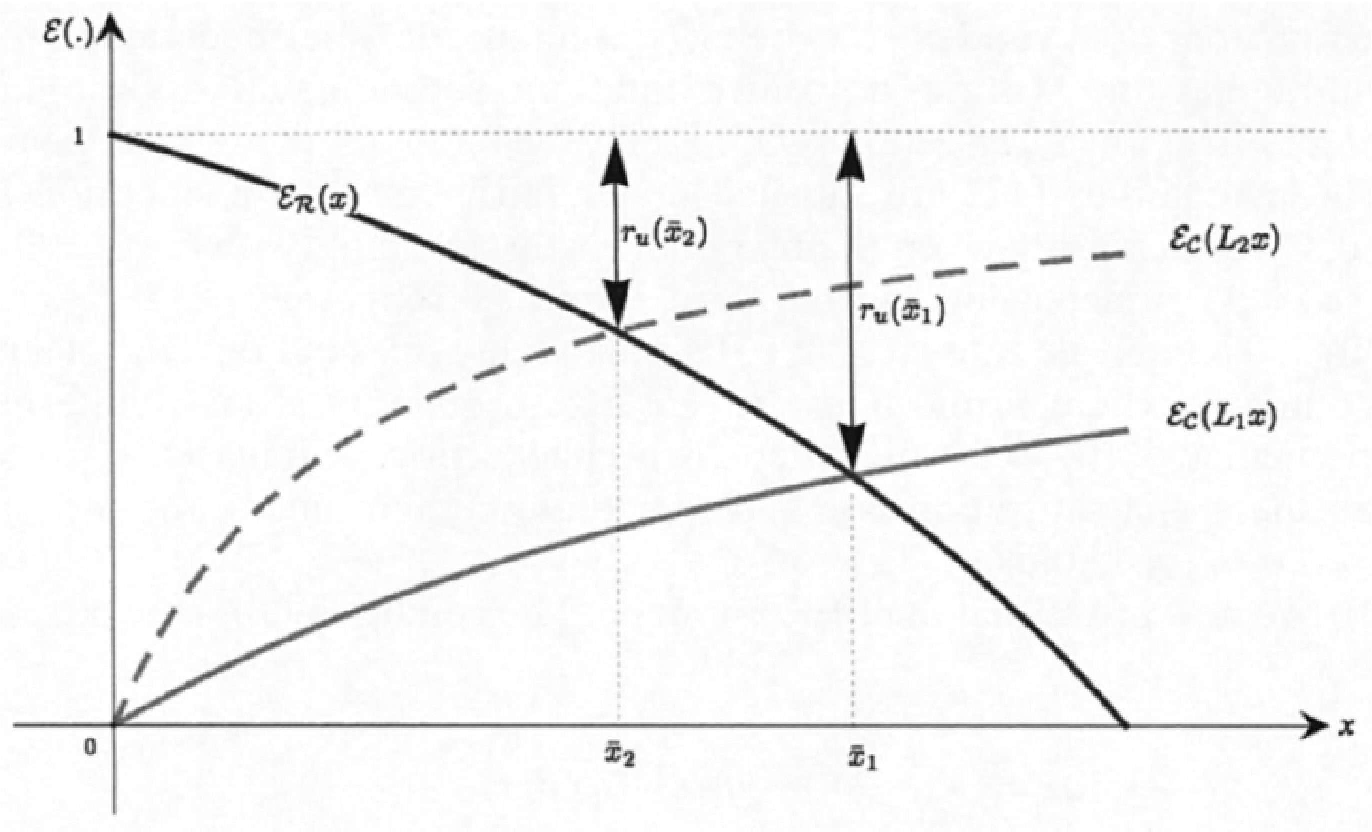

FIGURE 1.——Impact of L on the FEE under increasing RLV.

3.2 Market Size

A

Consumption per capita

$$\frac{d\bar{x}}{dL} < 0$$

An increasing RLV leads to a mild decline in consumption.

Output

B

$$\bar{q} = L\bar{x} \qquad \frac{d\bar{q}}{dL} > 0$$

3.2 Market Size

C

Markup and Price

$$\bar{M} = r_u(\bar{x})$$

$$\frac{d\bar{M}}{dL} < 0$$

$$\bar{p} = \frac{C'(q)}{1 - r_u()\bar{x}}$$

$$\frac{d\bar{p}}{dL} < 0$$

Number of Varieties

D

$$\bar{N} = \frac{EL}{C(\bar{q})}$$

$$\frac{d\bar{N}}{dL} > 0$$

3.2 Market Size

- ## PROPERSITION 2

The properties of the market outcome are determined by the variety-loving attitude of consumers and not by the production conditions.

Elasticity	$r'_u(\bar{x}) > 0$	$r'_u(\bar{x}) = 0$	$r'_u(\bar{x}) < 0$
Price $\bar{p}(L)$	$\mathcal{E}_{\bar{p}} < 0$	$\mathcal{E}_{\bar{p}} = 0$	$0 < \mathcal{E}_{\bar{p}}$
Diversity $\bar{N}(L)$	$0 < \mathcal{E}_{\bar{N}} < 1$	$\mathcal{E}_{\bar{N}} = 1$	$1 < \mathcal{E}_{\bar{N}}$
Consumption $\bar{x}(L)$	$-1 < \mathcal{E}_{\bar{x}} < 0$	$\mathcal{E}_{\bar{x}} = -1$	$\mathcal{E}_{\bar{x}} < -1$
Output $\bar{q}(L)$	$0 < \mathcal{E}_{\bar{q}} < 1$	$\mathcal{E}_{\bar{q}} = 0$	$\mathcal{E}_{\bar{q}} < 0$

PART FOUR 4

Derivation under CES

Derivation under CES

ADD RELATED TITLE WORDS

The CES utility function:

$$u(x) = \frac{x^{\rho}}{\rho} \ (\rho \leq 1 \ and \ \rho \neq 0)$$

$$u'(x) = x^{\rho-1}, u''(x) = (\rho - 1)x^{\rho-2}$$

Under the CES, *the relative love for variety* (RLV):

$$r_u(x) = -\frac{xu''(x)}{u'(x)} = -\frac{x \cdot (\rho - 1)x^{\rho-2}}{x^{\rho-1}} = 1 - \rho$$

Derivation under CES

ADD RELATED TITLE WORDS

$$R = pq = \frac{Lxu'(x)}{\lambda}$$

$$\frac{dR}{dx} = \frac{L(u'(x) + xu''(x))}{\lambda}$$

The elasticity of $R(x)$:

$$\varepsilon_R = \frac{x}{R}\frac{dR}{dx} = \frac{u'(x) + xu''(x)}{u'(x)} = 1 + \frac{xu''(x)}{u'(x)} = 1 - r_u(x)$$

Under the CES:

$$\varepsilon_R = 1 - r_u(x) = 1 - (1 - \rho) = \rho$$

$$\varepsilon_u = \frac{x}{u}\frac{du}{dx} = \frac{x \cdot x^{\rho-1}}{\frac{x^\rho}{\rho}} = \rho$$

$$\varepsilon_R = \varepsilon_u$$

Derivation under CES

ADD RELATED TITLE WORDS

Elasticity	$r_u'(\bar{x}) > 0$	$r_u'(\bar{x}) = 0$	$r_u'(\bar{x}) < 0$
Price $\bar{p}(L)$	$\mathcal{E}_{\bar{p}} < 0$	$\mathcal{E}_{\bar{p}} = 0$	$0 < \mathcal{E}_{\bar{p}}$
Diversity $\bar{N}(L)$	$0 < \mathcal{E}_{\bar{N}} < 1$	$\mathcal{E}_{\bar{N}} = 1$	$1 < \mathcal{E}_{\bar{N}}$
Consumption $\bar{x}(L)$	$-1 < \mathcal{E}_{\bar{x}} < 0$	$\mathcal{E}_{\bar{x}} = -1$	$\mathcal{E}_{\bar{x}} < -1$
Output $\bar{q}(L)$	$0 < \mathcal{E}_{\bar{q}} < 1$	$\mathcal{E}_{\bar{q}} = 0$	$\mathcal{E}_{\bar{q}} < 0$

In the CES case:

$$r_u(\overline{x}) = 1 - \rho \Rightarrow \; r_u'(\overline{x}) = 0$$

Derivation under CES

ADD RELATED TITLE WORDS

$$\overline{q} = L\overline{x}$$

$$\frac{d\overline{q}}{dL} = \overline{x} + L\frac{d\overline{x}}{dL} = \overline{x}\left(1 + \frac{L}{\overline{x}}\frac{d\overline{x}}{dL}\right) = \overline{x}(1 + \varepsilon_{\overline{x}/L})$$

$$when\ \frac{d\overline{q}}{dL} > 0\,, -1 < \varepsilon_{\overline{x}/L} < 0$$

$$when\ \frac{d\overline{q}}{dL} > 0\,, \varepsilon_{\overline{x}/L} < -1$$

In the CES case, $\frac{d\overline{q}}{dL} = 0$ and thus:

$$\varepsilon_{\overline{x}/L} = -1$$

Derivation under CES

ADD RELATED TITLE WORDS

$$\overline{P} = \frac{C'(\overline{q})}{1 - r_u(\overline{x})}$$

In the CES case, $r_u(\overline{x}) = 1 - \rho$, $\frac{d\overline{q}}{dL} = 0$ and thus:

$$\overline{P} = \frac{C'(\overline{q})}{\rho}$$

$$\frac{d\overline{P}}{dL} = \frac{C''(\overline{q})}{\rho}\frac{d\overline{q}}{dL}$$

$$\varepsilon_{\overline{P}} = \frac{L}{\overline{P}}\frac{d\overline{P}}{dL} = \frac{LC''(\overline{q})}{C'(\overline{q})}\frac{d\overline{q}}{dL} = 0$$

Derivation under CES

ADD RELATED TITLE WORDS

$$\overline{N} = \frac{LE}{C(\overline{q})}$$

$$\frac{d\overline{N}}{dL} = \frac{E}{C(\overline{q})} - \frac{LEC'(\overline{q})}{C(\overline{q})^2}\frac{d\overline{q}}{dL}$$

$$\varepsilon_{\overline{N}} = \frac{L}{\overline{N}}\frac{d\overline{N}}{dL} = \frac{C(\overline{q})}{E}\left(\frac{E}{C(\overline{q})} - \frac{LEC'(\overline{q})}{C(\overline{q})^2}\frac{d\overline{q}}{dL}\right) = 1 - \frac{LC'(\overline{q})}{C(\overline{q})}\frac{d\overline{q}}{dL}$$

In the CES case, $\frac{d\overline{q}}{dL} = 0$ and thus:

$$\varepsilon_{\overline{N}} = 1$$

Derivation under CES

ADD RELATED TITLE WORDS

Summary

1. The CES is the only function for which entry does not impact the equilibrium price.
2. Under the CES, equilibrium size of firms is independent of the market size.
3. Under the CES, individual welfare always increases with market size.

PART FIVE 5

EXTENSIONS

Multisector Economy

a two-sector economy

Heterogeneous Firms

firms with different cost

Nonadditive Preferences

the utility function is nonadditive

1

Utility function

$$\max_{X,Y} U(X,Y) = U\left[\int_0^N u(x_i)\,di, Y\right]$$

2

Budget Constraint

Choosing the unit of the homogeneous good for the marginal productivity of labor to be equal to 1 and choosing the homogeneous good as the numéraire, the equilibrium wage is equal to 1. E is endogenous

$$\int_0^N p_i x_i\,di + Y = E + Y = 1$$

EXTENSIONS

Multlsector Economy

3 For any given E

$$\max_{x_i \geq 0} \int_0^N u(x_i)\, di$$

$$\text{s.t.} \int_0^N p_i x_i\, di = E$$

4 Symmetry

$$v(p, N, E) \equiv Nu\left(\frac{E}{Np}\right)$$

5 The question is transformed into

$$\max_E U[v(p, E, n), 1 - E]$$

EXTENSIONS

Multlsector Economy

6

For sufficient conditions, utilities U and u yield an expenditure function E(p,N) that satisfies the properties

$$0 \le \frac{p}{E} \cdot \frac{\partial E}{\partial p} < 1 \qquad \frac{N}{E} \cdot \frac{\partial E}{\partial N} < 1$$

7

It is proved that the equilibrium mass of varieties increases with L when the three inequilities above hold.

The firm's cost function is given by $V(q,\theta)$ and $V(q,\theta)$ is strictly increasing in θ for all $q>0$. The parameter θ is distributed according to the continuous density $\gamma(\theta)$ defined on $[0,\infty)$

Firms face the same inverse demand function $p_\theta(x_\theta) = u'(x_\theta)/\lambda$. Spence-Mirrlees condition implies that lower θ leads to greater output, lower price and higher profit.

$$\overline{M}_\theta = r_u(\bar{x}_\theta) = 1/\sigma(\bar{x}_\theta)$$

$$\pi_0^*(\theta, \lambda; L) \equiv \max_{q \geq 0} \left\{ \frac{u'(q/L)}{\lambda} q - V(q, \theta) \right\}$$

Cut-off efficiency index

$$\pi_0^*(\overline{\theta}, \lambda; L) - F = 0$$

The free entry condition can be written as

$$\int_0^{\overline{\theta}(\lambda; L)} [\pi_0^*(\theta, \lambda; L) - F] \gamma(\theta) d\theta - F_e = 0$$

The partial derivative of π_0* with respect to L

$$\frac{\partial \pi_0^*}{\partial L} + \frac{\partial \pi_0^*}{\partial \theta}\frac{d\bar{\theta}}{dL} + \frac{\partial \pi_0^*}{\partial \lambda}\frac{d\bar{\lambda}}{dL} = 0$$

Transform

$$\varepsilon_{\bar{\theta}} = \frac{r(\bar{q}_{\bar{\theta}}/L) - \varepsilon_{\bar{\lambda}}}{\bar{\theta} \cdot \frac{\partial V(\bar{\theta})}{\partial \theta}}$$

$$\int_0^{\bar{\theta}} [r_u(\bar{q}_{\bar{\theta}}/L) - r_u(\bar{q}_{\theta}/L)]\,\bar{R}(\theta)\gamma(\theta)\,d\theta$$

$$\varepsilon_{\bar{\theta}} = \frac{r(\bar{q}_{\bar{\theta}} / L) - \varepsilon_{\bar{\lambda}}}{\bar{\theta} \cdot \frac{\partial v(\bar{\theta})}{\partial \theta}}$$

$$\int_0^{\bar{\theta}} [r_u(\bar{q}_{\bar{\theta}} / L) - r_u(\bar{q}_{\theta} / L)] \bar{R}(\theta) \gamma(\theta) d\theta$$

subutility function

utility gained from consuming x_i

$$u(x_i, X) = x_i - \frac{x_i^2}{2} - \gamma x_i \int_0^N x_j \, dj$$

inverse demand function

linear U and Lagrange multiplier equals 1

$$p(x_i, X) = 1 - x_i - \gamma X$$

RLV

price-increasing regime

$$r_u(\bar{x}) = \frac{\bar{x}}{\bar{x} + c}$$

EXTENSIONS

Nonadditive Preferences

demand for variety i

translog expenditure function

$$d(p_i;\Lambda_{trans},L)=\frac{L}{p_i}(\Lambda_{trans}-\beta\ln p_i)$$

constant absolute risk-aversion function

$$d(p_i;\Lambda_{cara},L)=\frac{L}{p_i}(\Lambda_{cara}-\beta\ln p_i)$$

equlibrium price

translog expenditure function

$$\beta(p-c)^2/p=cf/L$$

constant absolute risk-aversion function

$$\beta(p-c)^2/p=f/L$$

result

the market outcome under the nonadditive translog behaves like
the market outcome under the additive CARA utility