Universal Gravity

By Treb Allen, Costas Arkolakis, Yuta Takahashi
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小组成员：许力夕 胡杰 王婧 何昂 徐恺哲

Treb Allen

- Position: Dartmouth College, Department of Economics, Distinguished Associate Professor of Economics and Globalization
- Education
  - Ph.D. in Economics(with distinction), 2012 Yale University
  - M.Phil. in Economics, 2010 Yale University
  - M.A. in Economics, 2009 Yale University
  - B.A. in Political Economy (with honors), 2006 Williams College

Research Interests: Trade and Development

Costas Arkolakis

- Position: Department of Economics,
  Yale University, Professor of Economics
- Education
  - Ph.D. in Economics, 2007 University of Minnesota
  - M.A. in Economics, 2005 University of Minnesota
  - B.A. in Economics, 2001 Aristotle University of Thessaloniki, Greece
- Research interests: International Economics, Spatial Economics, Macroeconomics, Industrial Organization

Yuta Takahashi

- Position: Institute of Economic Research, Hitotsubashi University

- Education
  - Ph.D. in Economics, 2018 Northwestern University
  - M.A. in Economics, 2011 Kyoto University
  - B.A. in Economics, 2009 Kyoto University

- Research Interests : Macroeconomics and International trade

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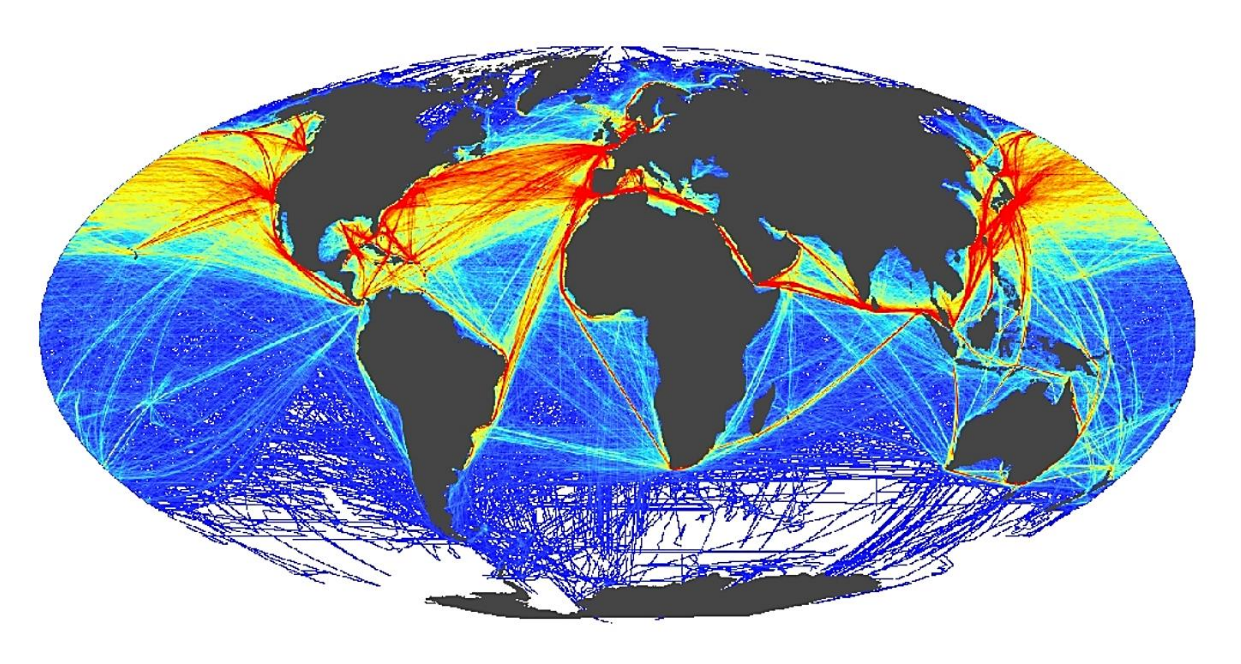
Motivation

- 经典贸易模型 （李嘉图模型、H-O模型等）： 无法进行实证研究

  原因：真实世界中存在（i）多个贸易国家（ii）双边贸易成本



全球海运航线分布图

来源：Ben Halpern, www.seaweb.org

- 贸易引力模型：

$$X_{ij} = c\frac{Y_i Y_j}{D_{ij}} \Longrightarrow logX_{ij} = logc + log\ Y_i + logY_j - logD_{ij}$$

理论基础不够扎实

Anderson and van Wincoop(2003) : “Estimated gravity equations do not have a theoretical foundation.”

- 引力方程理论建构：

The Armington model

“the first theoretical foundation for the gravity relationship.”

随着空间经济学的量化革命，大量理论模型产生。

复杂的假设和均衡不利于我们理解模型的本质性质：

均衡何时存在且唯一？

冲击的影响在不同模型中是否相同？

Universal Gravity Framework

从Armington gravity模型的两个变体开始

基本假定

I. 全世界有N个国家，每国生产一种不同的产品：$S \equiv \{1,2,\dots,N\}$

II. 每个国家的劳动力为L_i，$\sum_{i\in S} L_i = \bar{L}$

III. 产品从i地运送到j地有运输成本，即$\mathrm{p_{ij}} = p_i \cdot \tau_{ij}$
（其中p_i 是i地的产出价格，$\tau_{ij} \geq 1$）

IV. 消费者有*CES*形式的偏好

变体一：Trade Model

模型设定

I. 各国劳动力外生给定，并且完全无弹性 ⇒ （L_i 为固定常数）

II. 各国进行迂回式生产，生产函数满足CD形式:

$Q_i \equiv (A_i L_i)^{\xi} I_i^{1-\xi}$,其中$\xi \in (0,1]$

III. 产出价格$p_i = (\frac{w_i}{A_i})^{\xi} P_i^{1-\xi}$

IV. 消费者CES价格指数与中间投入品单位价格

$$P_i \equiv \left[\sum_{i \in S} (p_j \tau_{ji})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

变体二：Economic Geography Model

模型设定：

I.劳动力供给是完全弹性的，可以自由地在各国流动

⇒均衡时福利相同$\frac{w_i}{P_i}u_i = \frac{w_j}{P_j}u_j$

II.劳动生产率、舒适度均受溢出效应的影响⇒ $A_i = \overline{A_i}L_i^a$

$$u_i = \overline{u_i}L_i^b$$

III.生产函数为：$Q_i = \overline{A_i}L_i^{a+1}$

IV.产出价格为：$p_i = \frac{w_i}{\overline{A_i}L_i^a}$

两个变体的总需求

- 记i国与j国之间的贸易流为$X_{ij} = p_{ij}Q_{ij}$
- j国的总支出为$\mathrm{E}_j = \sum_{i=1}^{N} X_{ij} = \sum_{i=1}^{N} p_{ij}Q_{ij}$
- CES 消费者偏好$\Rightarrow \max \mathrm{U}_j = \left(\sum_i Q_{ij}^{\frac{1-\delta}{\delta}}\right)^{\frac{\delta}{1-\delta}}$

$$s.t. \sum_i p_{ij}Q_{ij} = E_j$$

$$\Rightarrow X_{ij} = \frac{(p_i\tau_{ij})^{1-\delta}}{\sum_{k\epsilon S}(p_k\tau_{kj})^{1-\delta}} E_j \quad \cdots\cdots(1)$$

两个变体的总供给

- 1.Trade Model
  - $Q_i = (A_i L_i)^{\zeta} I_i^{\,1-\zeta}$
  - $p_i = \left(\frac{w_i}{A_i}\right)^{\zeta} P_i^{1-\zeta}$
  - $P_i \equiv [\sum_{k\in S}(p_k \tau_{ik})^{1-\sigma}]^{\frac{1}{1-\sigma}}$

加上条件：$P_i\, I_i = L_i\, w_i$

$$\Longrightarrow Q_i = A_i L_i \left(\frac{p_i}{P_i}\right)^{\frac{1-\zeta}{\zeta}} \quad \cdots\cdots(2)$$

- 2.Economic Geography Model
  - $Q_i = \overline{A_i} L_i^{a+1}$
  - $u_i = \overline{u}_i L_i^b$
  - $A_i = \overline{A_i} L_i^a$
  - $p_i = \frac{w_i}{A_i L_i^a}$

加上均衡时的福利条件：$\frac{w_i}{P_i} u_i = \frac{w_j}{P_j} u_j$

$$\Longrightarrow Q_i = \kappa \overline{A_i}^{\frac{b-1}{a+b}} \overline{u_i}^{-\left(\frac{1+a}{a+b}\right)} \left(\frac{p_i}{P_i}\right)^{-\frac{1+a}{a+b}}, \kappa = \left\{\frac{\bar{L}}{\left[\sum_{i\in S}(\bar{A}_i \bar{u}_i)^{-\frac{1}{a+b}} \left(\frac{p_i}{P_i}\right)^{-\frac{1}{a+b}}\right]^{1+a}}\right\} \quad \cdots\cdots(3)$$

两个变体的共同均衡

市场出清：（总产出=总销售）

$Y_i \equiv p_i Q_i = \sum_{j\in S} X_{ij}$ ……(4)

贸易平衡：（总收入=总支出）

$Y_i \equiv p_i Q_i = E_i$……(5)

①&④&⑤&②or③ $\Longrightarrow \begin{cases} p_i^{1+\phi}\bar{c_i}\left(\frac{p_i}{P_i}\right)^{\psi} = \sum_{j\in S}\tau_{ij}^{-\phi}P_j^{\phi}p_j\bar{c_j}\left(\frac{p_j}{P_j}\right)^{\psi} & \text{……(6)} \\ P_i^{-\phi} = \sum_{j\in S}\tau_{ij}^{-\phi}p_j^{-\phi} & \text{……(7)} \end{cases}$

Trade model: $\psi \equiv \frac{1-\xi}{\xi}, \phi \equiv \sigma - 1$， $\bar{c_i} \equiv A_i L_i$

Economic Geography model: $\psi \equiv -\frac{1+a}{a+b}, \phi \equiv \sigma - 1$， $\bar{c_i} \equiv \bar{A}_i^{\frac{b-1}{a+b}}\bar{u}_i^{-\frac{1+a}{a+b}}$

$\bar{c_i}$是由i决定的外生变量,被称为$supply\ shifter$

需求弹性ϕ和供给弹性ψ则是由模型决定的参数⇒ 称为Gravity Constants（引力常数）

两个Armington模型的变体引入Universal Gravity Framework

$$p_i^{1+\phi}\overline{c_i}\left(\frac{p_i}{P_i}\right)^{\psi}=\sum_{j\in S}\tau_{ij}^{-\phi}P_j^{\phi}p_j\overline{c_j}\left(\frac{p_j}{P_j}\right)^{\psi} \quad \cdots\cdots(6)$$

$$P_i^{-\phi}=\sum_{j\in S}\tau_{ij}^{-\phi}p_j^{-\phi} \quad \cdots\cdots(7)$$

- 两个变体的均衡可以用相同的式子来表达均衡
- 两个变体的均衡是与它们的需求弹性和供给弹性紧密联系的

本文将提出一个由**六个条件**组成的框架来统一符合这些条件的贸易模型

Universal Gravity Framework

- 定义

符号	含义
Q_i	i国的产量
Q_{ij}	i国出口到j国的商品量
p_i	i国商品的产出价格
p_{ij}	i国出口到j国的商品的价格
$Y_i \equiv p_i Q_i$	i国产出商品的总价值
$X_{ij} = p_{ij} Q_{ij}$	i国到j国的贸易流
$E_i = \sum_{j \in S} X_{ji}$	i国进口商品的总价值,即i国总支出
$W_i \equiv {}^{E_i}/_{P_i}$	真实支出
$\frac{p_j}{P_j}$	真实产出价格
$P_i \equiv \left(\sum_{j \in S} p_{ji}^{-\phi}\right)^{-\frac{1}{\phi}}$	i国的价格指数

六个条件

- Condition1

$$p_{\mathrm{ij}} = p_i \cdot \tau_{ij} \cdots\cdots(8)$$

其中p_i 是i地的产出价格， τ_{ij}是贸易摩擦成本， $\tau_{ij} \geq 1$

- Condition2（总需求） $\Longrightarrow X_{ij} = \frac{(p_i\tau_{ij})^{-\phi}}{\sum_{k\epsilon S}(p_k\tau_{kj})^{-\phi}} E_j \cdots\cdots(9)$

其中$P_{\mathrm{j}} = \left[\sum_{i\in S}(p_{\mathrm{i}}\tau_{\mathrm{ij}})^{-\phi}\right]^{-\frac{1}{\phi}}, E_j = (\sum_{i\in S} p_{ij}^{-\phi})^{-\frac{1}{\phi}} W_j$

- Condition3(总供给） $Q_i = \kappa\overline{c_i} \, (\frac{p_j}{P_j})^{\psi} \cdots\cdots(10)$

其中$\overline{c_i}$&ψ&ϕ是外生给定的；κ是内生的

- Condition4（市场出清）

$$Y_i \equiv p_i Q_i = \sum_{j\in S} p_{ij} Q_{ij} = \sum_{j\in S} X_{ij} \qquad \cdots\cdots(11)$$

$$其中 Q_i = \sum_{j\in S} \tau_{ij} Q_{ij}$$

- Condition5（外生赤字）$\forall i \in S, E_i = \Xi\ \xi_i p_i Q_i$

Ξ是一个内生标量，为了确保世界市场出清需要有：

$$\Xi = \frac{\Sigma_i p_i Q_i}{\Sigma_i \xi_i p_i Q_i} \qquad \cdots\cdots(12)$$

当ξ_i=1且Ξ=1时，达到贸易平衡

- Condition6（标准化） $\sum_{i\in S} Y_i = 1$

均衡情况推导：

给定引力常数ϕ,ψ;

supply shifters $\overline{c_i}$;

双边贸易摩擦 τ_{ij}

结合C.1-C.5

$\Downarrow$

$$p_i^{1+\phi}\overline{c_i}\left(\frac{p_i}{P_i}\right)^{\psi} = \sum_{j\in S}\tau_{ij}^{-\phi}P_j^{\phi}p_j\overline{c_j}\left(\frac{p_j}{P_j}\right)^{\psi}$$

$$P_i^{-\phi} = \sum_{j\in S}\tau_{ij}^{-\phi}p_j^{-\phi}$$

$\Downarrow$

可得$\frac{p_i}{P_i}$

再将全球的总收入标准化为1，

C.6 $\sum_{i\in S}Y_i = 1$

$\Downarrow$ +C.4&C.5

可以求得$E_i = Y_i$

$\Downarrow$

可以求出X_{ij}

EXAMPLES OF MODELS IN THE UNIVERSAL GRAVITY FRAMEWORK

Model	Demand Elasticity (ϕ)	Supply Elasticity (ψ)	Model Parameters	Welfare of Labor
Armington 1969; Anderson 1979; Anderson and van Wincoop 2003 (with intermediates)	$\sigma - 1$	$(1-\zeta)/\zeta$	σ, substitution parameter; ζ, labor share	$B_i \times (p_i/P_i)^{1+\psi}$
Krugman 1980 (with intermediates)	$\sigma - 1$	$(1-\zeta)/\zeta$	σ, substitution parameter; ζ, labor share	$B_i \times (p_i/P_i)^{1+\psi}$
Eaton and Kortum 2002 (with intermediates)	θ	$(1-\zeta)/\zeta$	θ, heterogeneity parameter; ζ, labor share	$B_i \times (p_i/P_i)^{1+\psi}$
Melitz 2003; Di Giovanni and Levchenko 2013	$\theta[1 + \frac{\theta-(\sigma-1)}{\theta(\sigma-1)}]$	$(1-\zeta)/\zeta$	σ, substitution parameter; θ, heterogeneity parameter	$B_i \times (p_i/P_i)^{1+\psi}$
Allen and Arkolakis 2014	$\sigma - 1$	$-[(1+a)/(a+b)]$	σ, substitution parameter; a, productivity spillover; b, amenity spillover	$[\sum_i B_i(p_i/P_i)^{-1/(a+b)}]^{-(a+b)}$
Redding 2016	θ	$\alpha\varepsilon/[1+\varepsilon(1-\alpha)]$	θ, heterogeneity (goods) parameter; ε, heterogeneity (labor) parameter; α, goods expenditure share	$[\sum_i B_i(p_i/P_i)^{\alpha\varepsilon}]^{1/\varepsilon}$
Redding and Sturm 2008 (a variant of Helpman 1998)	$\sigma - 1$	$\alpha/[(1-\alpha)(\sigma-1)-\alpha]$	σ, substitution parameter; α, share spent on goods	$(\sum_i B_i(p_i/P_i)^{\psi\phi})^{(1+\psi)/(1+\psi\phi+\psi)}$
Economic geography with intermediate goods and spillovers	θ	$\frac{1-\zeta}{\zeta} - \frac{1}{\zeta b+a}(\frac{a+\zeta}{\zeta})$	θ, heterogeneity (goods) parameter; ζ, intermediate good share; a, productivity spillover; b, amenity spillover	$\sum_i B_i(p_i/P_i)^{1/\zeta}$
Economic geography with intermediate goods, idiosyncratic preferences, and spillovers	θ	$\frac{1-\zeta}{\zeta} + \frac{\varepsilon}{\zeta-\varepsilon(a+\zeta\beta)}(\frac{a+\zeta}{\zeta})$	θ, heterogeneity (goods) parameter; ε, heterogeneity (labor) parameter; ζ, intermediate good share; a, productivity spillover; b, amenity spillover	$\{\sum_i B_i[(\frac{p_i}{P_i})^{1/\zeta}]^{\varepsilon/\{1-\varepsilon[(a/\zeta)+b]\}}\}^{\{1-\varepsilon[(a/\zeta)+b]\}/\varepsilon}$

Note.—This table includes a (nonexhaustive) list of trade and economic geography models that can be examined within the universal gravity framework, the mapping of their structural parameters to the gravity constants, and the relationship between the welfare of workers and the real output prices. B_i is an exogenous, location-specific parameter whose interpretation depends on the particular model. λ is an endogenous variable that affects every country simultaneously.

通用引力框架内部解的存在性与唯一性

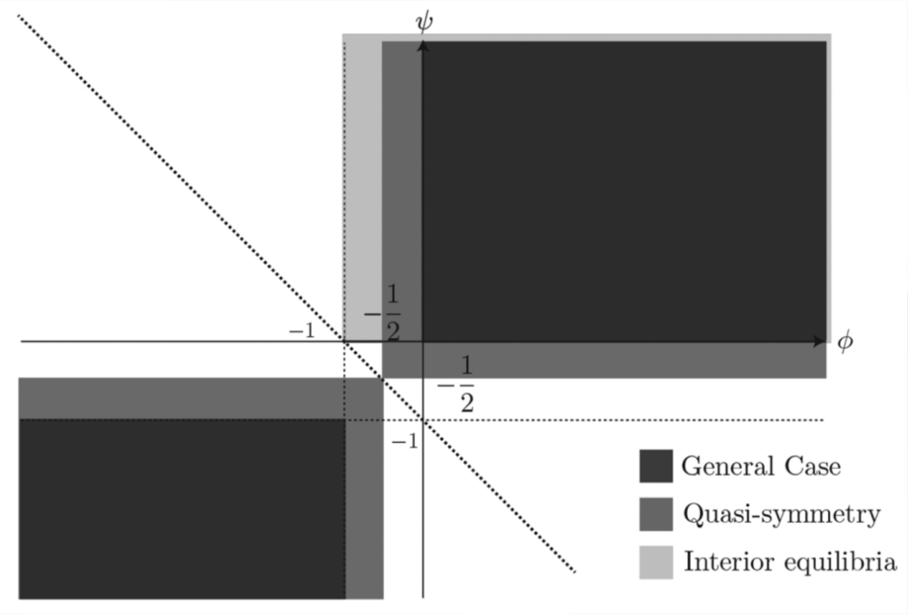
要解出*Universal Gravity Framework*均衡时解的性质，需要对贸易摩擦τ施加两个约束：

1） $\tau_{ii} < \infty, \forall i \in S$

2） $\{\tau_{ij}\}$ 矩阵的图是强联通的

$\Downarrow$

分析均衡时的性质不需要大量的参数
将条件简化为了两个引力常数：
需求弹性ϕ和供给弹性ψ



内点解存在与唯一的充分条件

I. $if\ 1+\psi+\phi \neq 0 \Rightarrow$ 存在内点解

II. $if\ \phi \geq -1\ \&\ \psi \geq 0 \Rightarrow$ 所有均衡都是内点解

III. $\{\phi \geq 0\ , \psi \geq 0\}\ or\ \{\phi \leq -1\ , \psi \leq -1\}$
$\Rightarrow$存在唯一的内点解

或者在满足准对称性的条件下$\left\{\phi \geq -\frac{1}{2}, \psi \geq -\frac{1}{2}\right\}\ or\ \left\{\phi \leq -\frac{1}{2}, \psi \leq -\frac{1}{2}\right\}$时
$\Rightarrow$存在唯一的内点解

The Network Effects of a Trade Shock

Universal Gravity Framework下一个地区性的贸易摩擦如何通过贸易网络传导改变均衡时各个国家贸易额、收入以及真实产出

供给$\vec{Q}^S_{N\times1}(\boldsymbol{p},\boldsymbol{P}) \equiv \left[p_i\bar{c}_i\left(\frac{p_i}{P_i}\right)^{\psi}\right]_{i\in S}$ ……(13)

需求$\vec{Q}^d_{N\times1}(\boldsymbol{p},\boldsymbol{P},\Xi,\tau) \equiv \left[\sum_{j\in S}\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}p_j\bar{c}_i\left(\frac{p_i}{P_i}\right)^{\psi}\Xi\xi_j\right]_{i\in S}$ ……(14)

价格向量 $\boldsymbol{p} \equiv (p_i)_{i\in S}, \boldsymbol{P} \equiv \left[(\sum_{j\in S}\tau_{ij}^{-\phi}p_j^{-\phi})^{-1/\phi}\right]_{i\in S}, \boldsymbol{\tau} \equiv (\tau_{ij})_{i,j\in S}$

在**均衡**状态下：总需求=总供给

即：$\vec{Q}^S_{N\times1}(\boldsymbol{p},\boldsymbol{P}) = \vec{Q}^d_{N\times1}(\boldsymbol{p},\boldsymbol{P},\Xi,\tau) \Leftrightarrow \sum_{i\in S}(1-\Xi\xi_i)p_j\bar{c}_i\left(\frac{p_i}{P_i}\right)^{\psi} = 0$

记$Z(\boldsymbol{p},\boldsymbol{P},\Xi)$=$\sum_{i\in S}(1-\Xi\xi_i)p_j\,C_i\left(\frac{p_i}{P_i}\right)^{\psi}$

贸易冲击$\boldsymbol{D}\boldsymbol{ln}\boldsymbol{\tau}$对均衡的影响表达式：

$$\left(\underbrace{\begin{pmatrix} D_{\ln\mathbf{p}}\mathbf{Q}^s & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{I} & 0 \\ \mathbf{0} & \mathbf{0} & D_{\ln\Xi}Z \end{pmatrix}}_{\equiv\mathbf{S}} - \underbrace{\begin{pmatrix} D_{\ln\mathbf{p}}\mathbf{Q}^d & D_{\ln\mathbf{P}}\mathbf{Q}^d - D_{\ln\mathbf{P}}\mathbf{Q}^s & D_{\ln\Xi}\mathbf{Q}^d \\ D_{\ln\mathbf{p}}\ln\mathbf{P} & \mathbf{0} & 0 \\ -D_{\ln\mathbf{p}}Z & -D_{\ln\mathbf{P}}Z & 0\end{pmatrix}}_{\equiv\mathbf{D}}\right)\begin{pmatrix} D\ln\mathbf{p}\\ D\ln\mathbf{P}\\ D\ln\Xi\end{pmatrix} = \underbrace{\begin{pmatrix} D_{\ln\boldsymbol{\tau}}\mathbf{Q}^d \\ D_{\ln\boldsymbol{\tau}}\ln\mathbf{P} \\ \mathbf{0}\end{pmatrix}}_{\equiv\mathbf{T}} D\ln\boldsymbol{\tau}$$

结合（13）&（14）两式，可以将$\boldsymbol{S},\boldsymbol{D},\boldsymbol{T}$化为关于引力常数的形式：

$$\mathbf{S} = \begin{pmatrix} (1+\psi)\mathbf{Y} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{I} & 0 \\ \mathbf{0} & \mathbf{0} & -\sum_{i\in S} E_i \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} -\phi\mathbf{E} + (1+\psi)\mathbf{X}, & (\phi-\psi)\mathbf{X} + \psi\mathbf{E}, & (E_i)_i \\ \mathbf{E}^{-1}\mathbf{X}^T & \mathbf{0} & 0 \\ (1+\psi)(Y_i - E_i)_i^T & -\psi(Y_i - E_i)_i^T & 0 \end{pmatrix} \quad \mathbf{T} = \begin{pmatrix} -\phi(\mathbf{X}\otimes\mathbf{1})\circ(\mathbf{I}\otimes\mathbf{1}) \\ \left(\mathbf{E}^{-1}\mathbf{X}^T\otimes\mathbf{1}\right)\circ(\mathbf{1}\otimes\mathbf{I}) \\ \mathbf{0} \end{pmatrix}$$

同时定义$\boldsymbol{A} \equiv \boldsymbol{S} - \boldsymbol{D}$

定理：对于任何$Universal\ Gravity\ Framework$中包含的模型而言，
假设$\boldsymbol{X}$满足强连通性，$\boldsymbol{A}$的秩为$2N$，则有如下三条性质：

$I.$产出价格和产出价格指数对贸易冲击的弹性为：

$$\frac{\partial \ln p_l}{\partial \ln \tau_{ij}} = -\phi X_{ij} A_{l,i}^{-1} + \frac{X_{ij}}{E_j} A_{l,N+j}^{-1} \qquad \frac{\partial \ln P_l}{\partial \ln \tau_{ij}} = -\phi X_{ij} A_{N+l,i}^{-1} + \frac{X_{ij}}{E_j} A_{N+l,N+j}^{-1}$$

$II.$如果$\boldsymbol{S}^{-1}\boldsymbol{D}$的所有特征值的绝对值都小于1，则$\boldsymbol{A}^{-1}$可以表示为：

$$\mathbf{A}^{-1} = \sum_{k=0}^{\infty} \left(\mathbf{S}^{-1}\mathbf{D}\right)^k \mathbf{S}^{-1}$$

$III.$如果贸易摩擦矩阵满足准对称性，且贸易平衡，且$\phi, \psi \geq 0$，
则对于所有的$i, l \in S, j \neq i, l$：

$$\frac{\partial \ln(p_i/P_i)}{\partial \ln \boxed{\tau_{il}}}, \frac{\partial \ln(p_l/P_l)}{\partial \ln \tau_{li}} < \frac{\partial \ln(p_j/P_j)}{\partial \ln \tau_{il}}$$

$$\frac{\partial \ln(p_i Q_i/P_i)}{\partial \ln \tau_{il}}, \frac{\partial \ln(p_l Q_l/P_l)}{\partial \ln \tau_{li}} < \frac{\partial \ln(p_j Q_j/P_j)}{\partial \ln \tau_{il}}$$

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估计引力常数 ϕ, ψ

利用隐含在模型中的工具变量

$“model - implied” \ instrument \ variables, MIIV$

$I.$ 理论→实证

理论	变量	实证数据来源
$X_{ij} = \frac{(p_i \tau_{ij})^{1-\sigma}}{\sum_{k \epsilon S}(p_k \tau_{kj})^{1-\sigma}} E_j$	X_{ij} $X_{ii} = X_i - \sum_{j \neq i} X_{ij}$ $Y_i = p_i Q_i = \sum_j X_{ij} = E_i$	*The Global Trade Analysis Project*
$Q_i = \kappa \bar{c}_i \left(\frac{p_i}{P_i}\right)^{\psi}$	$X_i^c \rightarrow \{\bar{c}_i\}$	*Nunn and Puga* (2012) *UNESCO* (2015) *Coe, Helpman, and Hoffmaister* (2009)
$p_{ij} = p_i \cdot \tau_{ij}$	τ_{ij}	*CEPII*数据集 *Head, Mayer, and Ries* (2010) 国家间距离

$II.$定义:

相对贸易量: X_{ij}/X_{jj}

相对总产出（总收入）: Y_i/Y_j

相对国内支出份额: $\lambda_{\mathrm{jj}}/\lambda_{ii}$,其中$\lambda_{jj} \equiv X_{jj}/E_j = \left[\tau_{jj}\left(\frac{p_j}{P_j}\right)\right]^{-\phi}$

$III.$ 模型变换

相对贸易量: X_{ij}/X_{jj}

$$X_{ij} = \frac{\left(p_i \tau_{ij}\right)^{1-\sigma}}{\sum_{k \epsilon S}\left(p_k \tau_{kj}\right)^{1-\sigma}} E_j \Longrightarrow \frac{X_{ij}}{X_{jj}} = \left(\frac{\tau_{jj} p_j}{\tau_{ij} p_i}\right)^{\phi}$$

使用$p_i = \frac{Y_i}{Q_i}, Q_i = p_i \overline{c_i} \left(\frac{p_i}{P_i}\right)^{\psi}$ 替代p_i:

$$\frac{X_{ij}}{X_{jj}} = \left[\frac{\tau_{jj}(Y_j - \overline{c_j})}{\tau_{ij}(Y_i - \overline{c_i})}\right]^{\phi} \left[\frac{\lambda_{jj} {\tau_{jj}}^{\phi}}{\lambda_{ii} {\tau_{ii}}^{\phi}}\right]^{\psi}$$

再取对数:

$$\boxed{In \frac{X_{ij}}{X_{jj}}} = -\phi In \frac{\tau_{ij}}{\tau_{jj}} + \phi \boxed{In \frac{Y_j}{Y_i}} + \psi \boxed{In \frac{\lambda_{jj}}{\lambda_{ii}}} - \phi In \frac{\overline{c_j}}{\overline{c_i}} + \phi\psi In \frac{\tau_{jj}}{\tau_{ii}}$$

$IV.$回归

回归方程: $\boxed{In\frac{X_{ij}}{X_{jj}}}=-\phi In\frac{\tau_{ij}}{\tau_{jj}}+\phi \boxed{In\frac{Y_j}{Y_i}}+\psi \boxed{In\frac{\lambda_{jj}}{\lambda_{ii}}}-\phi In\frac{\overline{c_j}}{\overline{c_i}}+\phi\psi In\frac{\tau_{jj}}{\tau_{ii}}$

存在内生性问题，无法使用OLS

$\tau_{ij},\overline{c_i}$ 无法观测，但与$\frac{Y_j}{Y_i},\frac{\lambda_{jj}}{\lambda_{ii}}$相关

解决方法:

(非参)估计$In\frac{\tau_{ij}}{\tau_{jj}}$: $-\phi In\frac{\tau_{ij}}{\tau_{jj}}=\beta_{cd}^{l}+\varepsilon_{ij}$

控制$In\frac{\overline{c_j}}{\overline{c_i}}$:添加控制变量X_i^c

$V.\text{“}model-implied\text{”}\ IV$

$$In\frac{X_{ij}}{X_{jj}} = -\phi In\frac{\tau_{ij}}{\tau_{jj}} + \phi In\frac{Y_j}{Y_i} + \psi In\frac{\lambda_{jj}}{\lambda_{ii}} - \phi In\frac{\bar{c_j}}{\bar{c_i}} + \phi\psi In\frac{\tau_{jj}}{\tau_{ii}}$$

$$\Longrightarrow In\frac{X_{ij}}{X_{jj}} = -\phi In\frac{\tau_{ij}}{\tau_{jj}} - In\pi_i + In\pi_j$$

其中$In\pi_i = \phi InY_i + \psi In\lambda_{ii} - \phi In\bar{c_i} + \phi\psi In\tau_{ii}$

构建虚拟世界——对$\tau_{ij}, \bar{c_i}$ 的估计值即为该世界实际情况

$\beta_{cd}^{l} \longrightarrow \tau;\ X_i^c \longrightarrow \{\bar{c_i}\}; \phi, \psi$ (Eaton and Kortum, 2002)

计算虚拟世界均衡下双边贸易及Y_i^*，λ_{ii}^*

$$In\pi_i = \phi In\underset{\Uparrow\ Y_i^*}{Y_i} + \psi In\underset{\Uparrow\ \lambda_{ii}^*}{\lambda_{ii}} - \phi In\bar{c_i} + \phi\psi In\tau_{ii}$$

Table 2: Estimating the gravity constants

	(1) OLS	(2) IV	(3) IV	(4) IV	(5) IV	(6) IV	(7) IV
Log Income (Demand	-0.403**	1.484	3.278	4.364*	3.882**	3.539***	3.715***
elasticity)	(0.171)	(1.157)	(2.674)	(2.371)	(1.838)	(1.356)	(1.312)
Log Own Expenditure	3.381**	92.889***	108.592**	116.649**	71.859**	64.968**	68.488**
Share (Supply Elasticity)	(1.600)	(13.417)	(48.104)	(47.944)	(35.883)	(33.014)	(32.198)
Land controls	No	No	Yes	Yes	Yes	Yes	Yes
Geographic controls	No	No	No	Yes	Yes	Yes	Yes
Historical controls	No	No	No	No	Yes	Yes	Yes
Institutional controls	No	No	No	No	No	Yes	Yes
Schooling and R&D controls	No	No	No	No	No	No	Yes
First stage Sanderson-Windmeijer F-test:							
Income		25.909	3.994	6.349	20.095	34.198	25.763
(p-value)		0.004	0.102	0.053	0.007	0.002	0.004
Own expenditure share		72.702	4.388	4.923	3.561	4.577	5.205
(p-value)		0.000	0.090	0.077	0.118	0.085	0.071
Observations	94	94	94	94	94	94	94

Notes: The dependent variable is the estimated country fixed effect of a gravity regression of the log ratio of bilateral trade flows to destination own trade flows on categorical deciles of distance variables, where the coefficient is allowed to vary by continent of origin and destination. Hence, each observation in the regressions above is a country. Instruments for income and own expenditure share are the equilibrium values from a trade model where the bilateral trade frictions are those predicted from the same gravity equation and countries are either identical in their supply shifters (column 2) or their supply shifters are estimated from a regression of observed income on observables (columns 3 through 7). In the latter case, the observables determining the supply shifters are controlled for directly in the first and second stage regressions, so identification of the demand and supply elasticities arise only from the general equilibrium effect on income and own expenditure shares. Land controls include land area interacted with fraction fertile soil, desert, and tropical areas. Geographic controls include the distance to nearest coast and the fraction of country within 100 km of an ice free coast. Historical controls include the log population in 1400 and the percentage of the population of European descent. Institutional controls include the quality of the rule of law. Schooling and R&D controls are average years of schooling (from UNESCO) and the R&D stocks (from Coe et al. (2009)), where a dummy variable is included if the country is not in each respective data set. Land, geographic, and historical control are from Nunn and Puga (2012). Standard errors clustered at the continent level are reported in parentheses. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01.

τ_{ji} τ_{ij}

缅甸 日本 泰国 中国 美国 法国 意大利 老挝

i j

零阶	对中美的影响	S^{-1}
一阶	对中美的伙伴国的影响	$(S^{-1}D)^{1}S^{-1}$
二阶	对中美的伙伴国的伙伴国的影响	$(S^{-1}D)^{2}S^{-1}$
⋮		
K阶	$(S^{-1}D)^{k}S^{-1}$	

τ_{ji}　　τ_{ij}

缅甸　日本　泰国　**中国**　**美国**　法国　意大利　老挝

i　　j

零阶　对中美的影响

在美国，美国产品变便宜，中国产品变贵

在中国，中国产品变便宜，美国产品变贵

$$\frac{p_i}{p_{ij}} = \text{real } p$$

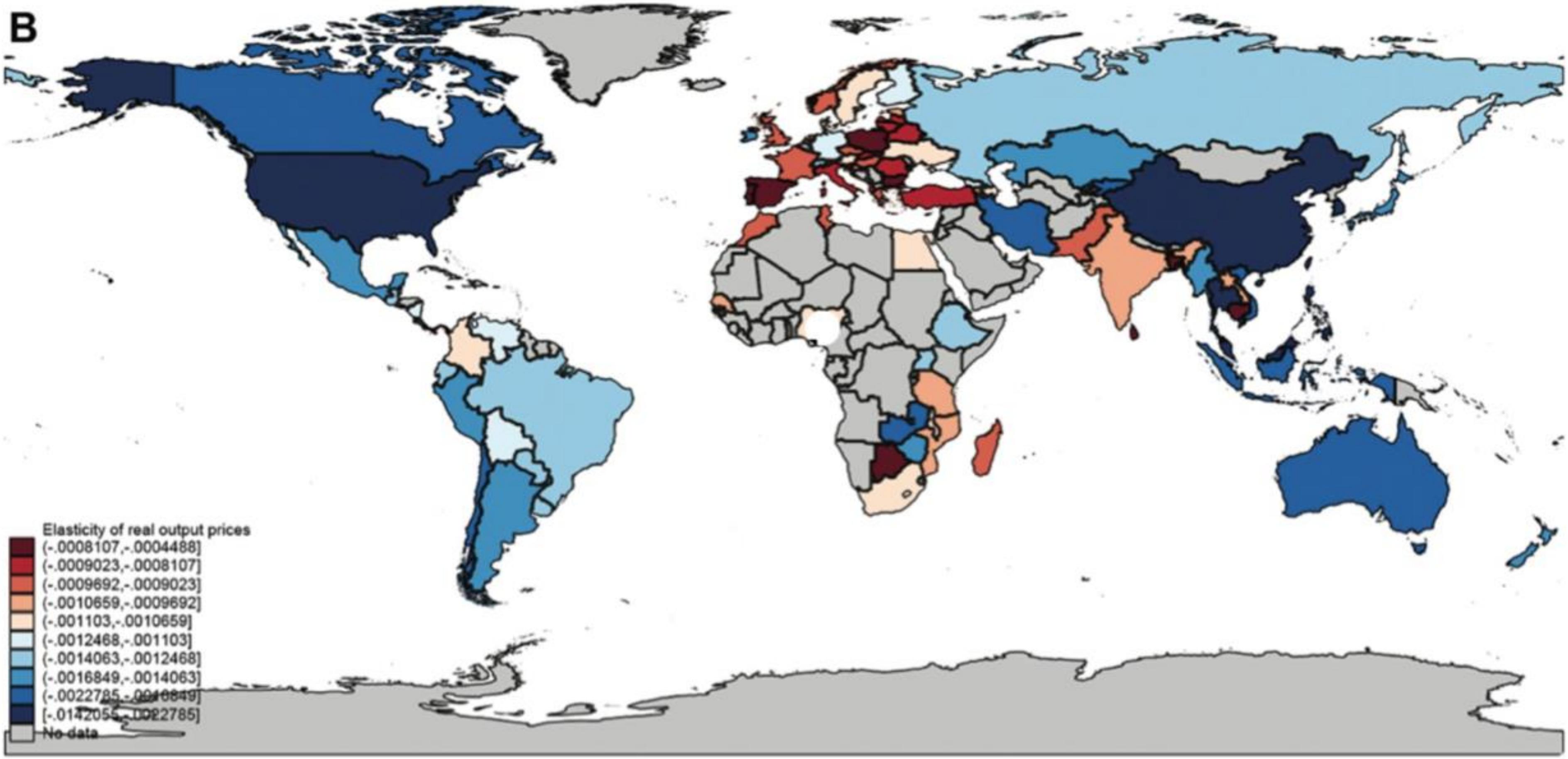
τ_{ij}

缅甸 日本 泰国 中国 美国 法国 意大利 老挝

i j

一阶 对伙伴国的伙伴国的影响

在美国和泰国，国内产品变便宜 $\frac{p_i}{p_{ij}}$ =real p


B
Elasticity of real output prices
(-.0008107,-.0004488]
(-.0009023,-.0008107]
(-.0009692,-.0009023]
(-.0010659,-.0009692]
(-.001103,-.0010659]
(-.0012468,-.001103]
(-.0014063,-.0012468]
(-.0016849,-.0014063]
(-.0022785,-.0016849]
[-.0142055,-.0022785]
No data

$\tau \uparrow 10\%$

中 real p ↓ 0.04% 美 real p ↓ 0.14%

因为供给弹性较大

中 E ↓ 2.7% 美 E ↓ 9.8%

因为需求弹性较小

两个顾虑

1、10%的shock过于微小，是否会造成结果不准确？ → 改为50%发现相关系数为0.99

2、随弹性系数不同，估计出来的影响会不会差异巨大？ → 进行置信区间分析

THANKS