Logistic Distribution

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1 Introduction

The Logistic distribution is a continuous probability density function that is symmetric and uni-modal. It is similar in appearance to the Normal distribution and in practical applications, the two distributions cannot be distinguished from one another.

2 Mathematical Definition

This distribution is characterized by two main parameters: location μ and scale σ. The probability density function is: (Raminta: I believe these signs are wrong. Do you agree?)

$$f(x) = \frac{1}{\sigma} * \frac{e^{-(x-\mu)/\sigma}}{1+e^{-(x-\mu)/\sigma)^2}}.$$

The cumulative distribution of the Logistic is the famous S-shaped curve on which Logistic regression is based:

$$F(x) = \frac{1}{1+e^{-(x-\mu)/\sigma}}.$$

3 Moments

The Compendium of the distributions (citation?) points out that the expected value is equal to the location parameter, μ:

$$E(x) = \mu$$

Because the distribution is symmetric, the median and the mode are also equal to α.
The variance of this distribution is:

$$Var(x) = \frac{1}{3}(\pi\sigma)^2.$$

Compare that value against the Normal distribution, $N(\mu,\sigma^2)$. The variance of the Logistic is different from the variance of the normal only by the scaling value of $\pi^2/3$

4 Illustrations

In Figure 1, some examples of Logistic distributions are offered.

In Figures 2 and 3, the density functions that correspond to various location and scale parameters are presented. If we look at Figure 2, we see that scale controls the width and the height of the distribution. As you see in Figure 3, if we vary the location of the density, the density curve shifts to the left and right, while the "spread" and "height" remains the same.

The Logistic density appears to be very similar to the Normal distribution. If we superimposed this distribution on the Normal, however, we would see that the Logistic has thicker tails. Consider Figure 4, in which a Logistic with location 0 and scale 1 is superimposed on a graph of the Normal density in which the variance is the same. That is to say, the variance of the Logistic is $\pi^2/3$ and the variance of the Normal is $\pi^2/3$ (or, equivalently, the standard deviation is $\pi/\sqrt{3}$).

Figure 5 illustrates the variations we see in the cumulative distribution function, when varying the scale parameter.

5 Conclusion

When would we use a logistic distribution? The Compendium says that is is often used instead "as an approximation to other symmetrical distributions due to the mathematical tractability of its CDF." Put in a more simple way, the Logistic gives a nice looking S-shaped curve with a relatively simple mathematical formula. The S-shaped curve, as seen in Figure 5, is used in the so-called Logistic regression model, which uses input variables to make predictions about how likelihood of certain outcomes. The S-shaped curve of the Logistic CDF is thought to be a substantively useful description of how the probability of an "event" or other outcome rises as a function of some input variables.

Figure 1: Logistic Variety

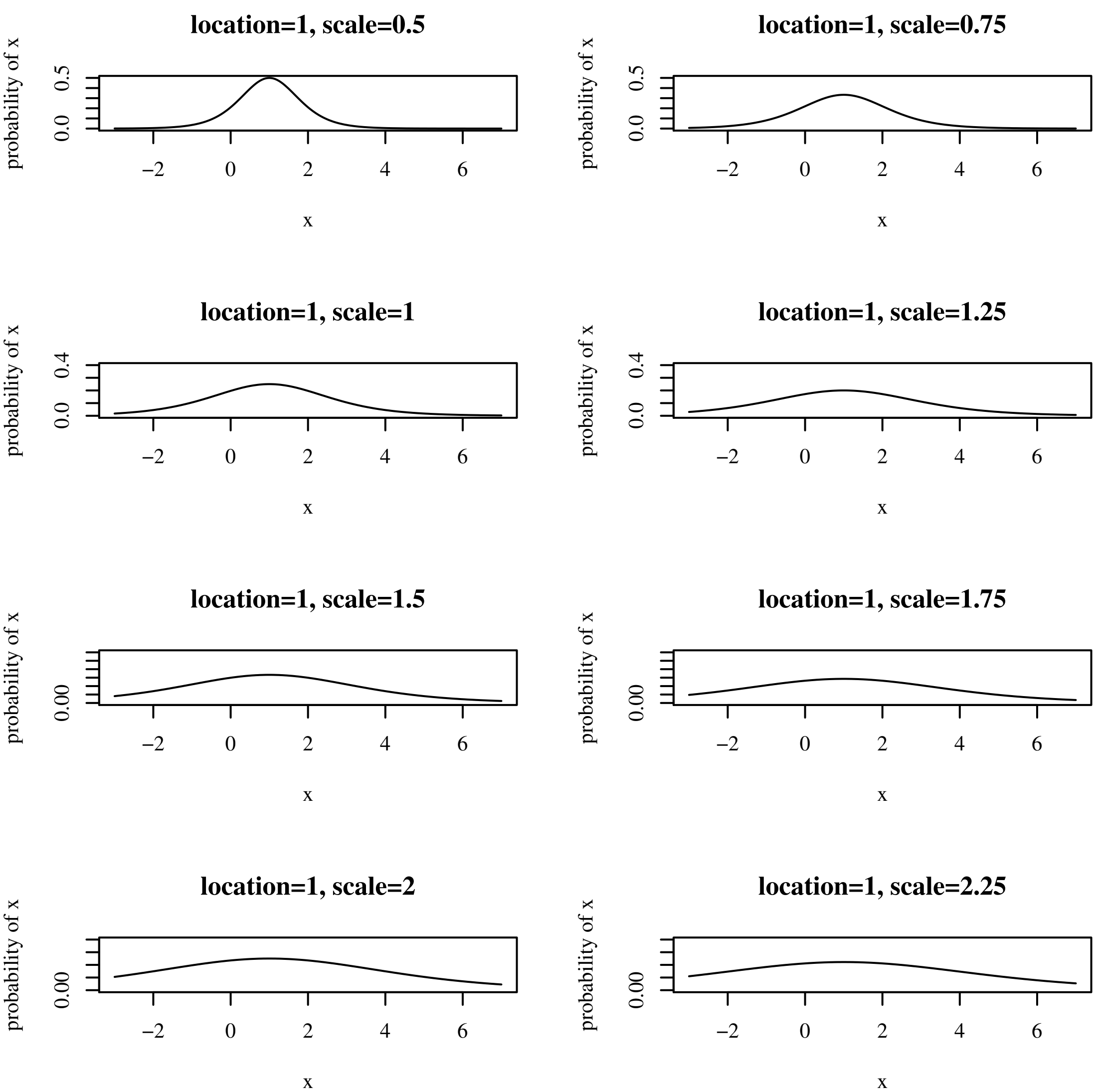

Figure 2: Logistic Distribution: Various Scales (location=0)

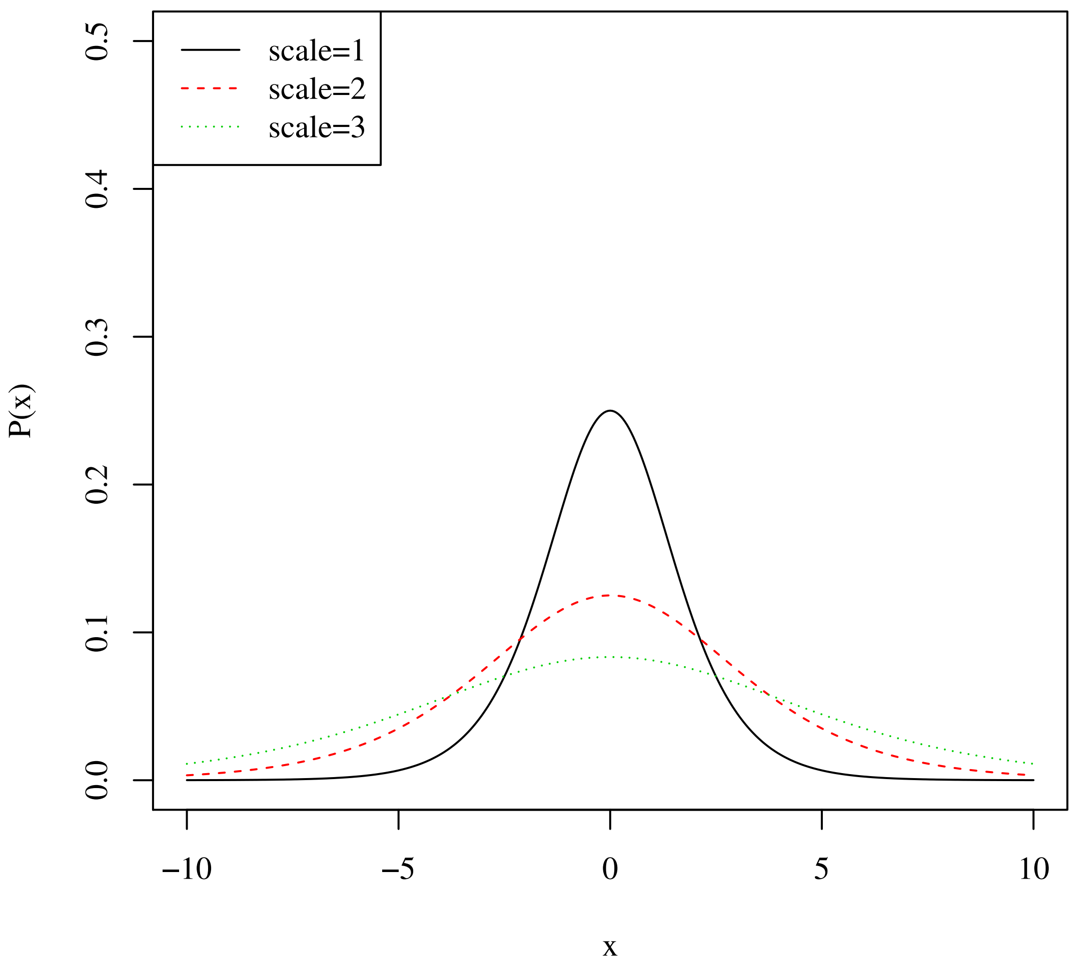

Figure 3: Logistric Distribution: Various Locations (scale=1)

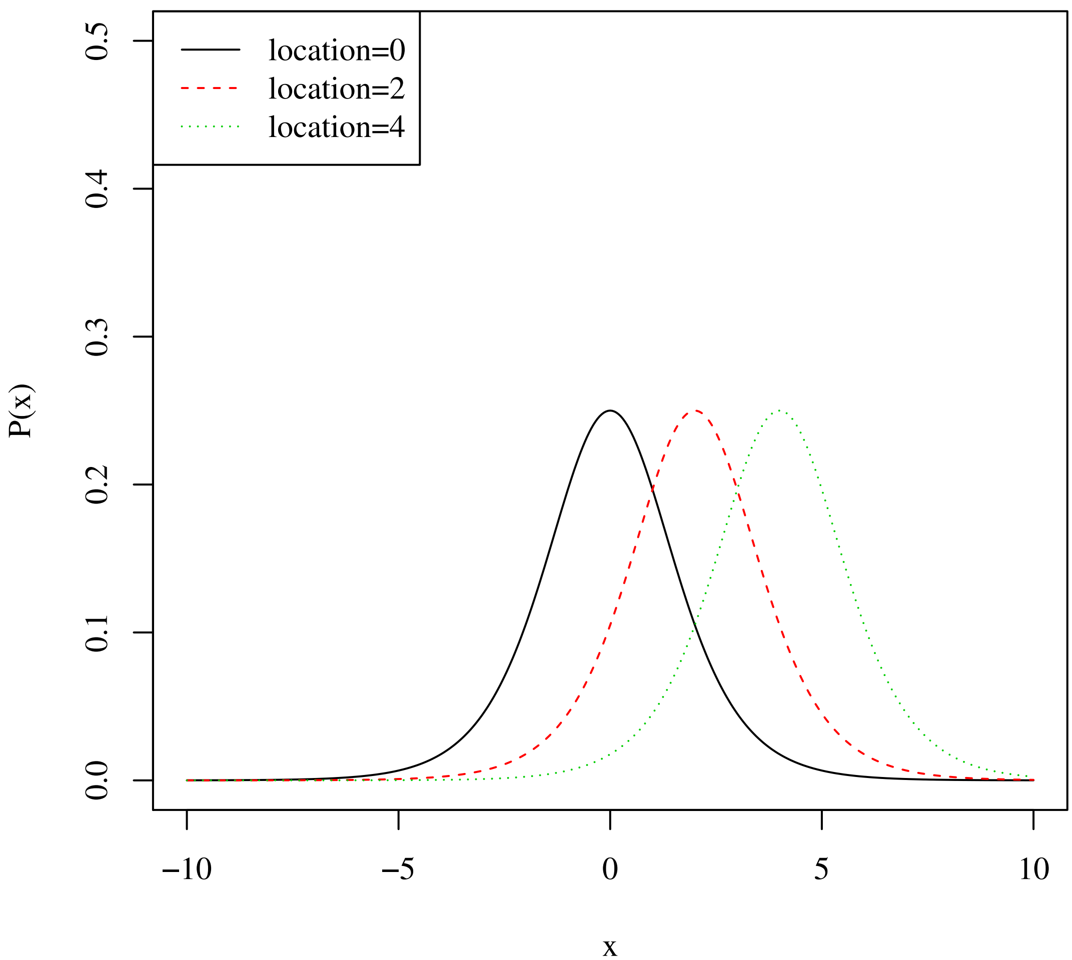

Figure 4: Logistric and Normal Distributions

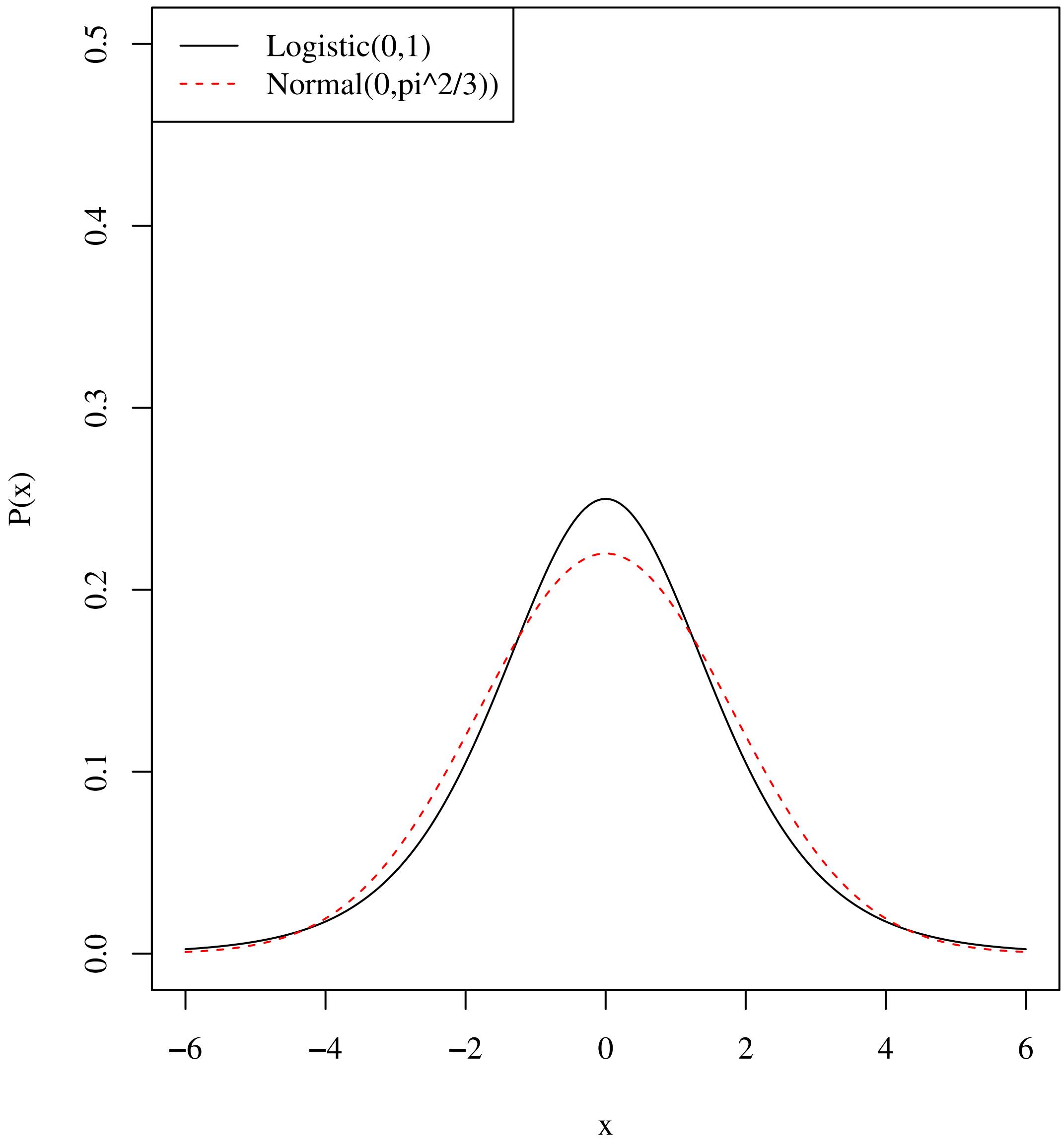

Figure 5: Cumulative Distribution of the Logistic